THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXVI JULY 1912 NUMBER 1

PHOTOGRAPHIC OBSERVATIONS OF COMET 1911 *c* (BROOKS)

By E. E. BARNARD

This comet was discovered by Dr. W. R. Brooks at Geneva, N.Y., on July 20, 1911. Perihelion passage occurred on October 27.

At first it was a faint object. Even with photographs of long exposure at this time there was no evidence of a tail. The comet was simply a round nebulous mass, a little brighter in the middle, but with no nucleus. It brightened rapidly, but was slow in developing a tail. By August 20 the comet was faintly visible to the naked eye as a hazy spot of light, but the tail was not seen until about September 16. It became a splendid object in the evening sky in the latter part of September, when its slender tail attained a length of about 25°. When near perihelion in the morning sky it was especially bright and beautiful. It was one of the finest comets of recent years and the duration of naked-eye visibility, about four months, has been exceeded by but few comets. At its best it was but little inferior to Halley's comet. Though a large one, it was not a great comet in the correct sense of the word. The regular parabolic rim outlining the head and tail was not formed. Even at perihelion small streamers at considerable angles to the tail were still present. These small streamers are usually absent in great comets, in which the outline of the head sweeps back to form the

boundaries of the tail when the comet makes a close approach to the sun, and all the small side streamers are swept back into the main tail.

THE PHOTOGRAPHS AND THE EXTRA GUIDING TUBE

The photographs were taken with the Bruce photographic telescope, the excellent mounting of which was made by Warner & Swasey. This instrument, as is well known (*Astrophysical Journal*, **21**, 35–48, 1905), is a triple telescope consisting of a visual guiding telescope with a Brashear object-glass of 5 inches aperture, and two portrait lenses, one of 10 inches diameter by Brashear, the other of 6.2 inches by Voigtländer (refigured by Brashear). A Clark 3.4-inch doublet is also attached to the mounting in a wooden camera-box. There is also attached to the instrument a Darlot lantern lens of 1.5-inch diameter, which will give a field 30° wide or more.

In the original condition of the mounting, the guiding eyepiece has a lateral motion of 1° or 2° with which a comet's head may be displaced by that amount from the center of the plate to show more of the tail. Having found that this displacement is not as much as the field of good definition will allow, I decided to attach an additional, light, adjustable tube to the mounting to secure more of a comet's tail. This tube will carry the object-glass and eye-end of the regular guiding telescope. It can be thrown out of the parallel of the main tubes for 6° or 8° in any direction. It will thus permit much more of a comet's tail to be shown by placing the head nearer the edge of the plate—still keeping within the field of fair definition. This tube can be placed in position on the mounting or removed in a few minutes' time. As this arrangement is wanted only when a large comet is present, it is very satisfactory, and the slight trouble of putting it in position is of no consequence. It has proved to be a very great advantage and was used in the present observations and in those of Halley's comet. This tube and its adjustments were made in the shop of the observatory by Mr. Oscar Romare.

All the photographs were made on Lumière Sigma plates.

Following are the approximate positions of the head of the comet

on the plates accompanying this paper taken from the *B.D.* charts for the epoch 1855.0.

Date	α	δ
September 22.....	$15^h\ 25^m$	$+52^\circ\ 29'$
October 19.....	12 39	+17 45
October 20.....	12 38	+16 20
October 23.....	12 33	+11 35
October 25.....	12 31	+ 8 42
October 27.....	12 30	+ 5 35
October 28.....	12 30	+ 4 10

Up to the middle of October the comet was a great disappointment from a photographic standpoint, for, though bright and conspicuous to the naked eye, its effect upon the sensitive plate was very slow, and long exposures were required to show the tail, which was simply a straight and rather narrow stream of diffused matter, with nothing of interest except occasionally an effort at additional streamers near the head. The photographs were very much alike night after night and it looked as if the comet would not be of any special importance. This condition continued until near perihelion, when it suddenly became an object of the deepest interest and showed a wealth of detail in the structure of the tail that has been equaled by only one comet (Morehouse's) since photography has recorded their spectacular changes. At the same time its photographic action increased very greatly, apparently much more than was due to its actual increase in brightness. Morehouse's comet, which was relatively a faint object, and just dimly visible to the naked eye for a couple of days, was not only remarkable for the great changes that took place in its phenomena, but was also singularly active in its effect on the photographic plate, requiring comparatively short exposures to show the full exten' of its tail, which was too faint even to be seen in the telescope. The spectroscope showed the presence of cyanogen strongly in this latter comet, and to this fact was attributed its great photographic activity and the unique features which developed in its tail.

The second, or active, period of Brooks's comet showed similar

characteristics of details and photographic effect, but, though cyanogen was present in the head, it was, so far as I can learn, not shown to be present in the tail. It might therefore be suggested that it was not really cyanogen that was responsible for the peculiarities of Morehouse's comet. Many of the phenomena resembled those of Morehouse's comet, and though not so weird in some of its features, it was on some occasions more beautiful than that object, from a photographic standpoint, and of course visually it was infinitely finer.

One fact that is strikingly apparent in the photographs of Brooks's comet is the remarkable reduction in the size of the head as it neared perihelion. On September 18 and 19 the actual diameter on the photograph was 1,200,000 km, while on October 28 it had diminished to 510,000 km.

At first the head was large and round, and the tail where it joined it was a narrow thread—a "neck"—as if the two had no more connection than a mere attachment. This is shown by the measures on September 18, when the head was 54′ in diameter and the "neck" only 6′. As the comet approached the sun, the head diminished in size, while the "neck" correspondingly increased, until finally the two became nearly of the same size, the outlines of the head more or less merging into and forming the outlines of the tail.

This phenomenon always occurs in the case of comets that make any near approach to the sun and is doubtless due to the increased pressure of the sun's light upon the matter of the comet which is thus driven out in the direction of the tail. But the photographic evidence of the phenomenon is a very beautiful one.

It would be difficult to describe the photographs of this comet obtained with the 10-inch lens, for they are very rich in detail. The scale of this lens was sufficiently great to show the structure of the tail and streamers in the best manner, and at the same time to secure enough of the length of the tail to exhibit its peculiarities to the best advantage. A brief effort at description, however, may serve to call attention to some of the features that might otherwise be overlooked. These descriptions refer to the photographs made with the 10-inch lens.

PLATE I

COMET 1911 *c* (BROOKS)

1911 September 22. $15^h\ 9^m$ G.M.T. $\alpha\ 15^h\ 25^m\ \delta + 52°\ 29'$ (1855.0). Exposure $3^h\ 30^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3′.4

PLATE II

COMET 1911 *c* (BROOKS)

1911 October 19. $22^h\ 38^m$ G.M.T. $\alpha\ 12^h\ 39^m\ \delta +17^\circ\ 45'$ (1855.0). Exposure $0^h\ 46^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3.′2

DESCRIPTIONS OF THE PHOTOGRAPHS

September 2. Head large and round. Tail near head very narrow.

September 13. Head large and round with very narrow "neck" from which the tail widens slightly.

September 15. Head large and round. "Neck" a mere thread from which the tail widens slightly.

September 16. Head large and round. "Neck" narrow. Tail widens slowly and consists of a brighter and a fainter portion.

September 18. Head large and round. "Neck" not quite so narrow as formerly. The tail double—a faint and a bright portion. The widest extension of head—very diffused and faint—is 54′; width of "neck" 6′.

September 19. Head large and round. "Neck" very slender. Tail widens slowly. On the north side is a slender ray 6° long.

September 21. Head large and round, extreme nebulosity 54′ in diameter. "Neck" narrow but not as narrow as before. A slender ray runs along the north side of the tail. A few irregularities in the tail.

September 22 (Plate I). In this photograph the tail is straight and slender and diffused, and where it joins the head it is very narrow. The head is very large and round, and from it also issue several incipient streamers.

Bad weather and moonlight interfered greatly after this, so that the views of the comet were few, and the photographs are of little interest until the latter half of October, when they are all that could be desired; but even here the series was badly broken by cloudy weather.

On October 19 (Plate II) the tail appears flat and widens out rapidly in the form of a nearly closed fan, and is bounded by a bright, sharply defined rim on each side, while on the body of the tail are many streaky details, some of which are curiously curved. Though the tail appears flat, it is doubtless a more or less thin shell of conical form. It is a striking and beautiful picture, and very much resembles the photograph of comet Morehouse on November 16, 1908.

October 20 (Plate III). The head is small and the tail rapidly widens out into a great bundle of diffused streamers. The effect is very curious and beautiful.

October 23 (Plate IV). The tail for some degrees from the head is wonderfully rich in fine threadlike streamers, nearly all of which are wavy in form, as if the tail had been combed with a coarse comb with an undulatory motion. At 2°.6 from the head a dark sinuous rift begins and extends beyond the limits of the plate. The head is scarcely any larger than the adjacent part of the tail. In the full extent of that portion of the tail shown, there is a remarkable amount of structural detail in the form of wavy streamers.

October 27 (Plate V). The head is very small. The north side of the tail is gently concave and sharply defined. There are a number of long irregular masses in it, about 6° or 8° from the head and many small streamers. This plate apparently shows the comet's tail fainter near the middle of its length. This peculiarity is verified by the 6-inch plate, but I question whether this is not due in some way to the clouds which covered the comet part of the time.

October 28 (Plate VI). This picture in some respects resembles that of October 20. The head is very small, apparently not any larger than the beginning of the tail. The tail widens out rapidly into a fan-shape with a central bright region. The small streamers near the head are wavy.

PHOTOGRAPHS TAKEN WITH THE LANTERN LENS

The following are descriptions of the tail as shown on the 1.5-inch lantern lens photographs. They are supplementary to the above.

October 23. At a point 19° from the head the tail suddenly widens from 2° to 3° on the north side. This continues for about 6°, until lost in space. There is another sudden widening of the tail on the north side about 10° from the head; this continues until the second widening occurs. The tail becomes bifurcated at 3°.7 from the head, the southern portion being composed of a slender ray, diffused on the south side. The total length of the tail on this plate is 26°.

PLATE III

COMET 1911 *c* (BROOKS)

1911 October 20. $22^h\ 44^m$ G.M.T. $\alpha\ 12^h\ 38^m\ \delta + 16°\ 20'$ (1855.0). Exposure $0^h\ 45^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3′.4

PLATE IV

COMET 1911 *c* (BROOKS)

1911 October 23. $22^h\ 32^m$ G.M.T. $\alpha\ 12^h\ 33^m\ \delta + 11^h\ 35^m$ (1855.0). Exposure $1^h\ 15^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3′.4

PLATE V

COMET 1911 *c* (BROOKS)

1911 October 27. $22^h\ 33^m$ G.M.T. $\alpha\ 12^h\ 30^m\ \delta + 5^\circ\ 35'$ (1855.0). Exposure $1^h\ 12^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3′.4

PLATE VI

COMET 1911 *c* (BROOKS)

1911 October 28. $22^h\ 38^m$ G.M.T. $\alpha\ 12^h\ 30^m\ \delta +4^\circ\ 10'$ (1855.0). Exposure $1^h\ 5^m$.
10-inch Bruce Telescope. Scale: 1 mm = 3′.4

LIST OF NEGATIVES OF BROOKS'S COMET

Date	Exposure	Quality			Length of Tail			Remarks
		10	6	3.4	10	6	3.4	
1911 July 26^{h} 11^{h} 20^{m}	2^{h} 50^{m}	G	G	—	0°	0°	—	Very difficult to guide on
Aug. 18 10 3	2 40	G	G	—	3	4	—	Mixed up with North America Nebula
21 9 25	1 15	G	G	—	Trace	1	—	
24 10 42	4 0	G	G	—	3–4	1	—	Sky whitish. Mixed up with North America Nebula
28 15 27	0 46	G	G	—	1	0	—	
31 12 52	4 15	G	G	—	4	6	—	Sky not v. transparent
Sept. 2 13 34	3 52	G	G	—	3	8	—	
13 8 9	1 40	G	P	—	6	6	—	Some clouds
14 9 6	1 14	P	F	—	6	8	—	Clouds
15 8 54	3 0	G	G	—	7	8	—	Sky fairly good
16 9 0	2 58	G	G	—	8	9	—	Sky dull
18 9 15	3 3	G	G	G	8	9	11	Clouds
19 8 31	2 10	G	G	G	8	5	11	Clouds
21 9 9	3 40	G	G	G	10½	12	11	
22 9 9	3 30	G	G	G	10½	12	12	Sky poor
23 7 46	1 1	F	F	F	6	12	11	Poor sky, and haze in last part
26 9 27	0 25	P	B	P	3	4	9	Clouds
27 8 59	2 4	G	—	F	10	—	12	10-in. neg. broken. Clouds
27 9 23	1 47	—	P	—	—	8	—	
29 9 29	2 10	G	F	P	11	14	12	Moonlight; poor sky
Oct. 7 6 58	0 32	B	B	—	2½	1–2	—	Full moon; sky poor
11 6 59	0 54	P	P	B	3–4	12	6	Haze
12 7 5	0 35	B	B	—	8–10	12	—	
19 16 38	0 46	G	G	G	9	12	8	Clouds
20 16 44	0 45	G	G	G	10	10	14	Sky poor
23 16 32	1 15	G	G	G	11	14	17	
24 16 58	0 33	P	G	P	3–4	—	6	Through breaks in clouds
25 16 38		—	—	F	—	—	15	Visible only once in a while through breaks in clouds
25 16 49	0 38	G	—	—	11	—	—	
25 16 51	0 42	—	G	—	—	14	—	
27 16 33	1 12	G	F	G	11½	14	15	In clouds most of time
28 16 38	1 5	G	G	G	11	14	15	Sky good

October 25. The tail gently widens and is better defined and slightly concave on the north side. Its length is 28°.

October 27. The tail is 30° long; it is slightly and irregularly concave and best defined to the north. Two condensations

occur in it at 7°.5 and 10°.5 from the head. The south side is diffused.

October 28. Length of tail 29°.5. Concave on north side, which is the best defined but irregular. One marked condensation (perhaps one of those of October 27) occurs at 15° from the head. Near the end on the north side the tail has several double curvatures. Unfortunately the reproduction of these plates (Plate VII) does not show the full extent of the tail as given on the photographs. The length of the tail on other lantern lens plates was as follows:

1911 September 16	11°	1911 September 27	22°
18	17	29	23
19	17	October 11	15
21	18	19	10
22	18	20	26
23	12		

Of course these different lengths do not mean that the tail varied, but for various reasons they represent the full extent shown on the plates.

On p. 7, not including the lantern lens pictures, is a list of the photographs which I have obtained with the three lenses of the Bruce photographic telescope. The headings 10, 6, 3.4, respectively, represent the 10-inch, 6-inch, and 3.4-inch lenses. In the columns of "Quality" and "Length of Tail" the initials mean B=bad, F=fair, G=good, and P=poor. A dash indicates that no photograph was secured with the particular lens.

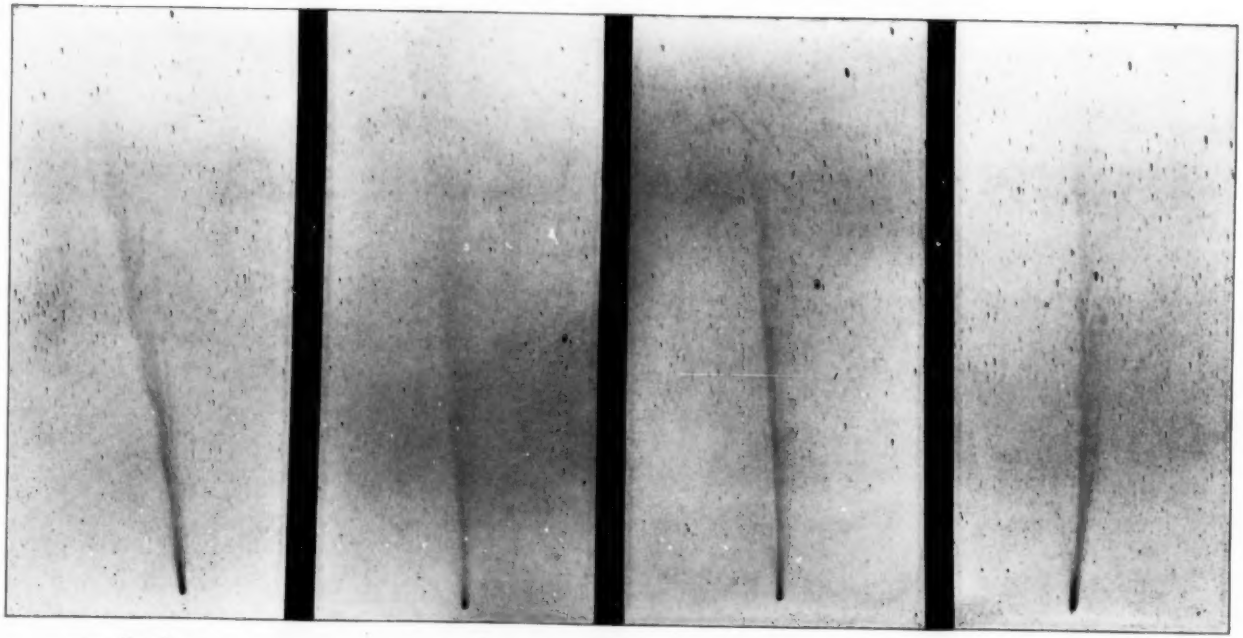
NOTES WITH THE 40-INCH TELESCOPE, ITS 4-INCH FINDER, THE 5-INCH BRUCE GUIDING TELESCOPE, AND WITH THE NAKED EYE

July 21, 10^h30^m. The central part is small, 0'.5 diameter, condensed, and 12.5 magnitude. From this condensed part the comet diffuses very softly beyond the limits of the field (6'). It is faintly visible in the 4-inch finder.

July 23, 14^h. It is very diffused and larger than the 6' field. The central condensation is 10" in diameter; no nucleus or tail.

July 25, 11^h30^m. Very large and very diffused, about 15' in diameter. The central condensation 10"–15" diameter and 12.5 magnitude. No nucleus or tail. Quite noticeable in finder and seems brighter than before.

PLATE VII



1911 October 23 | 1911 October 25 | 1911 October 27 | 1911 October 28

COMET 1911 *c* (BROOKS)

With the 1.5-inch lantern lens of the Bruce Telescope. Scale: 1 mm = 24′

July 30, 11^h. Very much larger, and somewhat brighter. With 700 diameters there is a small definite nucleus of 12.5 magnitude in a small inner nebulosity. Very much larger than the field of 460 (6′) and very diffused—except the condensed part. In finder it is 10′–15′ in diameter and quite conspicuous, but not much brighter in the middle.

August 6, 13^h. Very large and very diffused. Strong but small condensation to (perhaps) a nucleus of twelfth magnitude.

August 15, 12^h. Very large and diffused with small central condensation. Quite conspicuous in finder in spite of moonlight.

August 20, 10^h. Very much bigger than the field of view. Small central condensation of twelfth magnitude. In finder it is conspicuous and about 15′ in diameter and about 12.5 magnitude gradually condensed in the middle. It is faintly but distinctly visible to naked eye as a hazy spot of 5.5 or 6 magnitude.

August 21, 10^h. Five-inch guiding telescope of the Bruce photographic instrument. It is brighter, but no tail. With naked eye could not be certain of seeing it.

August 24, 12^h. Five-inch. Bright but no tail or nucleus.

August 25, 12^h. Very easy to naked eye as a small hazy spot 5^m or $5^m.5$.

August 28, 15^h30^m. Very easy to naked eye when looked for, as a small hazy spot of 5 magnitude. Exactly as noticeable as the two stars ω^2 and ω^3 *Cygni* (Proctor's Chart), *B.D.*+48°3142 ($4^m.9$) and *B.D.*+48°3145 ($5^m.9$), when they appeared as one. It was brighter than either, equaling the combined light of the two.

August 29, 12^h. It was conspicuous to the naked eye like a hazy 5^m star, but no tail was visible either to the naked eye or in the finder. With the 40-inch telescope the comet was several times larger than the field of view, with a small strong condensation, but no definite nucleus. This view was a very impressive one. I do not know any telescopic view of a comet that has produced such an impression of immensity. It seemed as if one were (as he really was) looking deep down into an enormous sphere of transparent vapor through which the distant stars were shining and in the center of which glowed a brighter condensation that illuminated it.

August 31, 12^h. Bright in 5-inch. No tail. To the naked eye

it was as noticeable as a 4^m–5^m star—a small round hazy spot. It was 1^m brighter than a small star very close east of it (+52°.2547, 5.3 magnitude).

September 1, 9^h30^m. It was bigger than the field of view. Perhaps a faint nucleus.

September 2, 15^h. To the naked eye about $4^m.5$, like a hazy star. No tail even in 5-inch.

September 3, 13^h. Very large with small condensation, but no tail or nucleus. Condensation very small, around it a fainter glow for 0'.5, which merges into the general soft glow extending away beyond the limits of the field.

Quite noticeable to naked eye as a 4^m–5^m star. It was equal in brightness to *B.D.*+49°.3062 ($4^m.9$). This star, however, is marked "variable."

September 10. Just before the moon rose, the comet was noticeable close north of *ξ Draconis*. It seemed inferior to the star—especially later when it was seen on a moonlit sky.

September 11, 10^h30^m. On a moonlit sky it was visible to the eye and appeared to be of the same brightness as *ξ Draconis* (3.5 magnitude).

September 13, 8^h. Very conspicuous to the naked eye, but unless looked at closely it would have been taken for a 4^m star. No tail. The light was quite intense, but the nebulosity very small. It was of the same brightness as *B.D.*+46°.2349 ($4^m.0$). It was as noticeable as *β Draconis* ($3^m.0$) but not so bright, perhaps a magnitude or more less than that star.

September 14, 9^h. In 5-inch, very bright and there seemed to be some tail preceding. To the naked eye it was of the same brightness as +46°.2349 ($4^m.0$). It was as noticeable as *β Draconis*, but 1^m or more less bright. When looked at carefully the nebulosity was very small. If glanced at casually one would readily pass it over as a star.

September 15, 9^h30^m. With naked eye it was close to a small star *B.D.*+57°.1702 (5.0 magnitude). The comet was very much brighter than this star. It was about the mean brightness of *ι Herculis* and *β Draconis*.

September 16, 10^h. Tail quite well seen in 5-inch, for, say, 1°,

but only noticeable close to comet. To the naked eye the comet was very noticeable, as much so as a third-magnitude star. It was about the mean brightness of *ι Herculis* and *β Draconis*. Faint tail perhaps seen with naked eye for 1° or more.

September 18, 10^h. The comet seemed to fluctuate in its light in the 5-inch while guiding on it.

It was very bright to the naked eye—like a small bright hazy star. It was almost the mean brightness of *ι Herculis* and *β Draconis*. Probably a faint tail for 3° or 4°, but not certain.

September 21, 10^h. With naked eye, could not be sure of any tail, but the comet itself was as bright as *ι Draconis* (3^m.0).

September 22, 7^{h}30^m. When I started to guide upon the comet with the 5-inch, a bright yellow star (+52°.1876) of the 7.0 magnitude was very slightly south from the center—about 15″. There must have been, a few minutes before, almost if not a perfectly central transit of the comet over the star. The star appeared like a bright yellow nucleus; by contrast the nebulosity of the comet was bluish white. The tail was noticeable in the telescope.

To the naked eye the comet was bright and small; could not be sure of any tail. At 8^h the comet was somewhat brighter than the star *ι Draconis*, *B.D.*+59°.1654 (3^m.0). It was bluish white and the light rather intense. The light of the star was more intense, but at a glance the comet was brighter than the star.

September 23, 8^h. To the naked eye it was brighter than +59°.1654. A faint tail 3° or 4° long was visible.

September 26, 9^h. To the naked eye the tail was 5° or 6° long and very faint.

September 27, 10^h. The tail could be traced about 20°. For one-half this distance it was very straight. Located on the chart it was 15°.5 long.

September 29, 9^h. Well seen in moonlight with faint tail. The head was brighter than third magnitude. After moonset the sky was thickish and the comet did not appear as bright as on September 27. The tail could be faintly traced for 15° or 20°.

October 11, 7^h. The head was noticeable to the naked eye, but the tail was dim; it (the tail) would pass 3° or 4° to the (right) north of *η Ursae Majoris*. It could be traced 6/10 (14°) the

distance to η. The head was much brighter than β *Canum Venaticorum* of 5.0 magnitude. The tail near the head was only fairly noticeable, and very dim the rest of the way.

October 12, 8^h. The sky was better and the comet brighter. The head perhaps as bright as the second magnitude. The tail was long and slender and noticeable. It would pass a couple of degrees to the left (south) of ζ *Ursae Majoris*. I could trace it nearly to that star, or over 25° in length.

October 17, 17^h. In the 5-inch there was a bright nucleus. The tail passed out of field preceding. To the naked eye the head was at least second magnitude and the tail could be traced for a few degrees, but the moon was too bright to make much out of it.

October 19, 15^h (in clouds). The nucleus was very bright in the 5-inch. No structure could be made out in that part of the tail in the field of view. The comet was very beautiful to the naked eye. The head was brighter than the first magnitude. The tail for 6° or 8° was pretty bright, but the rest of it was not striking. It seemed to broaden out, and pointed directly toward γ *Ursae Majoris*. The lower, or east, side of the tail passed 2° to the left of γ *Comae* (*B.D.*+29°2288) $4^m.7$ and could be traced for perhaps 10° beyond that star—about 24°. The sky was poor, being more or less misty.

October 20, 16^h30^m. To the naked eye the nucleus was starlike and brighter than the first magnitude. The tail for the first 3° or 4° was bright, then fainter. The left-hand (north) edge passed about a quarter of a degree or less to the right of γ *Comae*, and extended at least 10° farther, or about 25° in length. In the 40-inch telescope, at 17^h30^m, the nucleus was round and ill defined and not much brighter than the arc on which it was placed. Micrometric measures of the nucleus made its diameter $= 2''.24 = 1200$ km at 17^h50^m.

October 23, 16^h. Nucleus perhaps not so bright to the naked eye as before. It was not far from first magnitude, however. The tail, which was very long, seemed somewhat convex to the west. It could be traced to and slightly beyond *62 Leonis*, or about 25°. It passed over that star. The comet was a very conspicuous object. The tail for 10° or more was quite strong and farther out was rather faint and slender. It was a beautiful object when best seen.

In the 5-inch the head and body were slightly cone-shaped, the outline fairly definite, and very slightly brighter toward the axis. The nucleus was white, but not well defined.

October 24, 16^h30^m. Seen by glimpses through clouds. Nothing of importance.

October 25, 16^h30^m. Clouds, until daylight was killing the comet. The nucleus was much less bright, between second and third magnitude; the tail was long and straight.

To the naked eye the nucleus was just as bright as δ *Leonis* of third magnitude.

October 27, 17^h. In the 40-inch the nucleus was large and ill defined with a strong brushing out south following.

October 28, 16^h30^m. The comet was much less bright to the naked eye. The tail could be traced to β *Leonis*—about 18°—but was noticeable for only half the distance. Nucleus slightly less than δ *Virginis* ($+4°.2669$), $3^m.0$. Sky good.

FROM MT. WILSON, CALIFORNIA

November 13, 17^h. The comet was still visible to the naked eye with a tail 6° or 8° long. The nucleus was a little less bright than η *Corvi* (*B.D.*$-15°.3489$), $5^m.0$.

November 24, 17^h30^m. It could be seen faintly with the naked eye, and would have been brighter earlier in the night. The tail was faint, and the head was about 5.5 or 6 magnitude.

In reference to the pictures that accompany this paper, it may be interesting to add that the beautiful structures shown in the tail on the photographic plates in the latter part of October were not visible in any of the telescopes with which the comet was observed here. On several of the most interesting dates I had examined the head and the tail near it carefully to see if there was any structure that would show on the plates, but apparently there was none.

In several of the large plates accompanying this paper there are irregular defects of brightness on the sky, etc., that are not in the originals.

Micrometric measures of the position of the comet were made on a number of nights. These will be printed elsewhere.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
June 1, 1912

TERTIARY STANDARDS WITH THE PLANE GRATING[1] THE TESTING AND SELECTION OF STANDARDS

FIRST PAPER

By CHARLES E. ST. JOHN and L. W. WARE

INTRODUCTION

At the meeting, on Mount Wilson August–September 1910, of the International Union for Co-operation in Solar Research, the question arose as to the adaptability of the plane-grating spectrograph of long focus to interpolating between the secondary standards in the determinations of the standards of the third order, and in the measurement of arc, spark, and solar spectra in the international units. In the recommendations of the Committee on Wave-Lengths that were adopted by the Conference occur the following resolutions:[2]

> 4. The laboratories or observatories possessing first-rate concave gratings are invited to determine by interpolation as soon as possible standards of the third order in the spectrum of the iron arc within the above range of spectrum [i.e., λ 4282–λ 6494].
>
> 8. It is very desirable that in different laboratories possessing concave gratings of the best quality, photographs of arc, spark, and solar spectra, and new measurements according to the international system, shall be taken as soon as possible.

This preference of the concave grating over the plane grating was questioned by those who have had experience in the determinations of wave-lengths by means of the plane-grating spectrograph of long focus. The question was raised by Professor Plaskett, and in response the position of the Committee was given by Professor Kayser, who said:[3]

> I think it will be much more difficult to use plane gratings, because the spectrum is not normal and it is necessary to apply corrections. I have no plane gratings of my own and have never tried to measure wave-lengths with

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 61.

[2] *Transactions of the International Union for Co-operation in Solar Research*, **3**, 33.

[3] *Ibid.*, p. 42.

them, so I cannot tell if the order of precision is the same as for concave gratings, but in every case I should prefer concave gratings.

The discussion was later taken up by Newall, Adams, Fabry, and St. John,[1] and the point of view of those who have had experience was expressed by Newall, who said:

> Many of those who are engaged in solar observations are using plane gratings, arranged in the Littrow form, and many of us are of the opinion, rightly or wrongly, that the plane grating can compete quite well with the concave grating, provided the rest of the optical train is accurate enough. So it is hoped that provisionally we may regard the words "concave grating of the first quality" as not to be taken as of the essence of that resolution.

As is well known, considerable use of the plane grating is made at the observatory on Mount Wilson for determining the wave-length of spectral lines; and it is exceedingly important to know what degree of precision can be obtained by the use of the plane grating as compared with the concave grating in interpolating between standards about 50 Ångströms apart, when the former is employed in a spectrograph of great focal length.

The immediate motive for taking up the determination of the wave-lengths of the tertiary standards in the red arose from the need of standards for determining the wave-lengths of the oxygen band in the solar spectrum at about λ 6300. The original intention was to use the wave-lengths of the tertiary standards published by Professor Kayser,[2] but when these were tried it was at once found that they did not form a consistent series on Mount Wilson. If these tertiary standards were omitted, however, the secondary standards did form a consistent series, provided the international values for the wave-lengths were employed, instead of the "adjusted" values of Kayser. This induced us to redetermine the wave-lengths of some of the tertiary standards in this region, with what at first were surprising results. For this purpose we used the international standards of the second order. The results from a preliminary series of determinations are shown in Table I, together with the international standards and the corresponding data from Kayser's table of wave-lengths.

[1] *Ibid.*, pp. 65–66.

[2] *Astrophysical Journal*, **32**, 217, 1910.

It appears that the wave-lengths for the two lines λ 6297 and λ 6322 agree closely with those given by Kayser and come into almost exact agreement when the differences in the standards are taken into account, while for the three remaining lines the differences from Kayser's values are far outside the limits of error. It was at first exceedingly puzzling that lines in such close proximity to each other and to the standards, as are these five lines, should give values differing so widely from Kayser's. We soon convinced ourselves, however, that these were real differences and could not be attributed to errors of measurement. In seeking for an explanation, we compared our results with the behavior of the

TABLE I

International Standards	Kayser's Adjusted	Mt. Wilson	Kayser	Kayser—Mt. Wilson	Group	Pressure of 8 Atm. above Normal
6265.145	6265.145	.....	.....	0.000	*b*	0.070 Å.
.....	.....	6297.802	6297.804	+0.002	*b*	0.068
.....	.....	6301.519	6301.528	+0.009	*d*	0.25
.....	.....	6302.511	6302.522	+0.011	*d*	Similar to λ 6301 but unmeasurable
6318.028	6318.031	.....	.....	+0.003	*b*	0.080
.....	.....	6322.697	6322.698	+0.001	*b*	Group *b* by inspection
6335.341	6335.339	.....	.....	−0.002	*b*	0.074
.....	.....	6336.843	6336.850	+0.007	*d*	0.26
6393.612	6393.612	.....	.....	0.000	*b*	0.072

lines under pressure as found by Gale and Adams.[1] From their results we give in the sixth column of Table I the group to which the individual lines belong according to their classification, and in the seventh column the displacements to the red under a pressure of 8 atmospheres above normal. In the case of lines that could not be measured on these plates we give the results from inspection. It is in general easy to determine the group or the type of displacement by inspection, even when no measurements are possible. The line at λ 6302 shows on their plates a tremendous displacement to the red, similar to λ 6301, but too diffuse for measurement. It is at once apparent from inspection of the table that our

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 18, 1912.

results are related to theirs, and that the lines showing such tremendous displacements to the red under pressure on the laboratory plates are the lines showing a lessened wave-length on our plates. These lines belong to group *d* of their class 5, of which they say:[1]

> Almost all of the lines belonging to it occur in the yellow and red portions of the spectrum, and all show enormous displacements. The widening of these lines and their lack of symmetry are so great, however, that many of them are practically incapable of measurement, and for the others the degree of accuracy is extremely low. The measures when made are upon the maximum, which is always upon the violet side of the center of the line.

The first thought was that in the case of unsymmetrical lines the two types of gratings might conceivably give the lines a different appearance. The plane grating having no astigmatism reproduces the exact form of the lines, while the great astigmatism of the concave grating tends to obliterate the differences between the middle and the extremities of the lines, giving them a more uniform appearance throughout their length. This difference in the behavior of the two types of gratings, if effective, would tend, however, in the case of the plane grating to give a longer wave-length for lines so unsymmetrical on the red side, while our measurements gave shorter wave-lengths for these lines. It then occurred to us that the differences were of the order that the differences in atmospheric pressure between the Pasadena laboratory and the observatory on Mount Wilson would produce. The elevation of Mount Wilson is 5886 feet (1794 m), the mean barometric height being 25.4 inches (62.2 cm). The elevation of the Pasadena laboratory is about 800 feet (244 m), so that the difference of altitude corresponds to a change of approximately one-fifth of an atmosphere. Under this decrease of pressure the lines under consideration would undergo an absolute displacement of 0.006 Ångström to the violet, on the basis of the measurements of Gale and Adams. Considering the difficulty of the measurement of these plates, it seemed probable that the cause of the discrepancies would be found in the extreme sensitiveness of these lines to changes of pressure. As to the appear-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 15, 1912.

ance of these lines under a pressure of 8 atmospheres above normal, Gale and Adams say:[1]

> Group *d* is made up of a very few lines in the violet, a fair-sized group in the greenish-yellow, and a small number in the red, all of which show immense displacements. The lines are bright and widened enormously to the red, the wings extending sometimes from 5 to 10 Ångström units. The precision of measurement upon these lines necessarily is very low, in some cases the determination amounting to little more than an estimate of the maximum within the broad band.

The preliminary work seemed to promise interesting and important results, and it was then decided to pursue the investigation further. No one questions the adaptability of the plane grating to differential measurements, but in the plan here proposed the absolute wave-lengths were to be determined both on Mount Wilson and in the Pasadena laboratory. In this way the pressure-shifts would be determined by the differences, while for lines belonging to the same groups as the standards two independent determinations of wave-lengths would be obtained.

APPARATUS

The following investigations were made upon two series of plates:

1. A series taken on Mount Wilson with the 30-foot (9 m) Littrow spectrograph used in connection with the 60-foot tower telescope, in which a plane grating by Michelson was used. This grating has an available ruled surface of 2.25 inches (57.1 mm)× 4.88 inches (124 mm) and gives excellent definition for both bright and dark lines.

2. A series of plates made for us by Mr. Babcock with the 30-foot spectrograph of the Pasadena laboratory, in which another Michelson grating was used.

Thus the two series were made separately by two observers with two different instruments. In general the second order of the spectrum was employed, with a scale of approximately 1 mm =0.88 Ångström, but for the detailed investigation of particular regions the third order was employed in which the scale of the

[1] *Op. cit.*, pp. 31–32.

plates is approximately 1 mm=0.56 Ångström. The second order was used for the purpose of having a large number of standards upon each plate, and in order that the scale might be comparable with that used by Professor Kayser in obtaining his list of standards of the third order, since at first it was thought that the accuracy obtainable by means of the plane grating might be shown by a comparison of our results with those already published by Professor Kayser. Later it developed that a more convincing test of their precision could be made by a comparison of our results for Pasadena and the summit of Mount Wilson.

The arc used in both series is of the type suggested by Pfund,[1] in which a bead of iron oxide rests in a small cup-shaped bowl of the lower positive electrode consisting of an iron rod. The upper electrode is also of iron and carries a brass bushing to prevent overheating of the electrode. The arc was fed with a current of about 6 amperes from a 110-volt direct-current circuit. This type of arc proves to be very satisfactory in the steadiness of its burning, even for periods of an hour.

METHOD OF REDUCTION

The objection raised by the users of the concave grating to the employment of the plane grating for interpolating between standards is, of course, that the scale of the spectrum produced by the plane grating varies continuously along the plate. In practice the tilt of the photographic plate in the spectrograph of the Littrow type is such that it is tangent to the focal curve of the lens, and with an achromatic lens of 30 feet focus, plates 17 inches (43 cm) long may be made so nearly coincident with the curve for their whole length that the definition of the lines is excellent over the full extent of the plate. The scale is then so nearly a linear function of the wave-length that for runs of 50 to 60 Ångströms it is practically linear within the limits of measurement. In every case the scale of the plate at any point is given by the ordinates of a smooth curve whose abscissae are the wave-lengths of the spectral region covered by the plate. There can be no irregularities in the course

[1] *Astrophysical Journal*, **27**, 296, 1908.

of such a curve, as at each point it is a function of the corresponding point of the smooth color-curve of the lens employed, and of the angular distance from the middle of the plate. It is the practically linear character of the curve representing the scale of the plate that makes it possible to use the plane grating for interpolating between standards, and the inherent smoothness of the curve that makes it feasible to test the consistency of a series consisting of several standards. In the reduction of our plates the curve giving the scale of an individual plate was found by obtaining the factor (interval in Ångströms between two successive standards divided by the measured interval between the standards) for each two successive standards of the second order. These factors were carried to five significant figures, and plotted as ordinates with the mean wave-lengths of the corresponding intervals as abscissae. A smooth curve was drawn through the points, or sometimes straight lines were drawn from point to point, both methods yielding the same final results. Such curves are shown in Plate VIII, and will be discussed later in this paper.

In interpolating to obtain the wave-lengths of intermediate lines, only lines were considered which were situated between two standards on the plate under reduction; that is, lines near the ends of the plates and beyond the last standards at either end were not measured. To obtain such lines a series of overlapping plates was used. In calculating the wave-length of any line it was referred to the nearest standard of lesser and also to the nearest standard of greater wave-length. The reduction factors were read from the curve at the points whose abscissae were the means of the wave-lengths of the required line and the standards of lesser and greater wave-length, the approximate wave-length of the line required being known or previously obtained. It will be seen that the course of the curve at the points at which the factors are taken is influenced not alone by the ordinate of the curve fixed by the two standards directly involved, but by the standards on either side of those to which the required line is immediately referred. Hence, the wave-length of the intermediate line is based upon at least four standards and the agreement between the results of the forward and back interpolations furnishes a basis for judging of the consistency of

the standards involved, and the method binds the determination of the wave-length of the intermediate line to the four standards.

The measurement of the plates was done mainly upon a Toepfer comparator with a screw 300 mm long, and upon a smaller Toepfer comparator with a screw 150 mm long. Some plates, however, were measured upon Gaertner comparators with screws 80 mm long. In all cases the periodic errors were known to be negligible, and the errors of run were carefully investigated and the corrections applied to the measurements. The plates were measured red to right and red to left. At least four settings were made on each line in the two positions, bringing the cross-wire to the center of the line alternately from the right and left. In many instances, particularly in the case of the standards and some lines difficult to measure, the number of settings was doubled and sometimes tripled. Single and double cross-wires were employed, their use depending upon the intensity of the plates. In obtaining the dispersion-curve from the standards, it would be desirable to measure over the whole length of a plate without changing its position on the comparator. This we were not able to do even with the largest comparator, which required one break in the series. The conditions are less favorable as the range of the screws becomes shorter, and there is a great loss of time, as well as greater liability to error, in measuring overlapping sections of the plate in order to piece the sections together. In the latter case special care is needed in lining up the spectra and the screw. The error introduced by lack of parallelism between the spectra and the screw is not so serious in interpolating between two standards as in obtaining the distances between successive standards upon which the course of the dispersion-curve of the plate depends. A comparator is now under construction in the observatory instrument shop especially adapted to work of this kind, with which a plate 50 cm long can be measured without changing its adjustment. With this instrument it will be possible to obtain standards upon plates taken with higher dispersion, and accordingly to reach a higher degree of accuracy in obtaining a dispersion-curve for testing the consistency of a series of standards.

THE TESTING OF THE INTERNATIONAL STANDARDS

REGION λ 5975–λ 6494

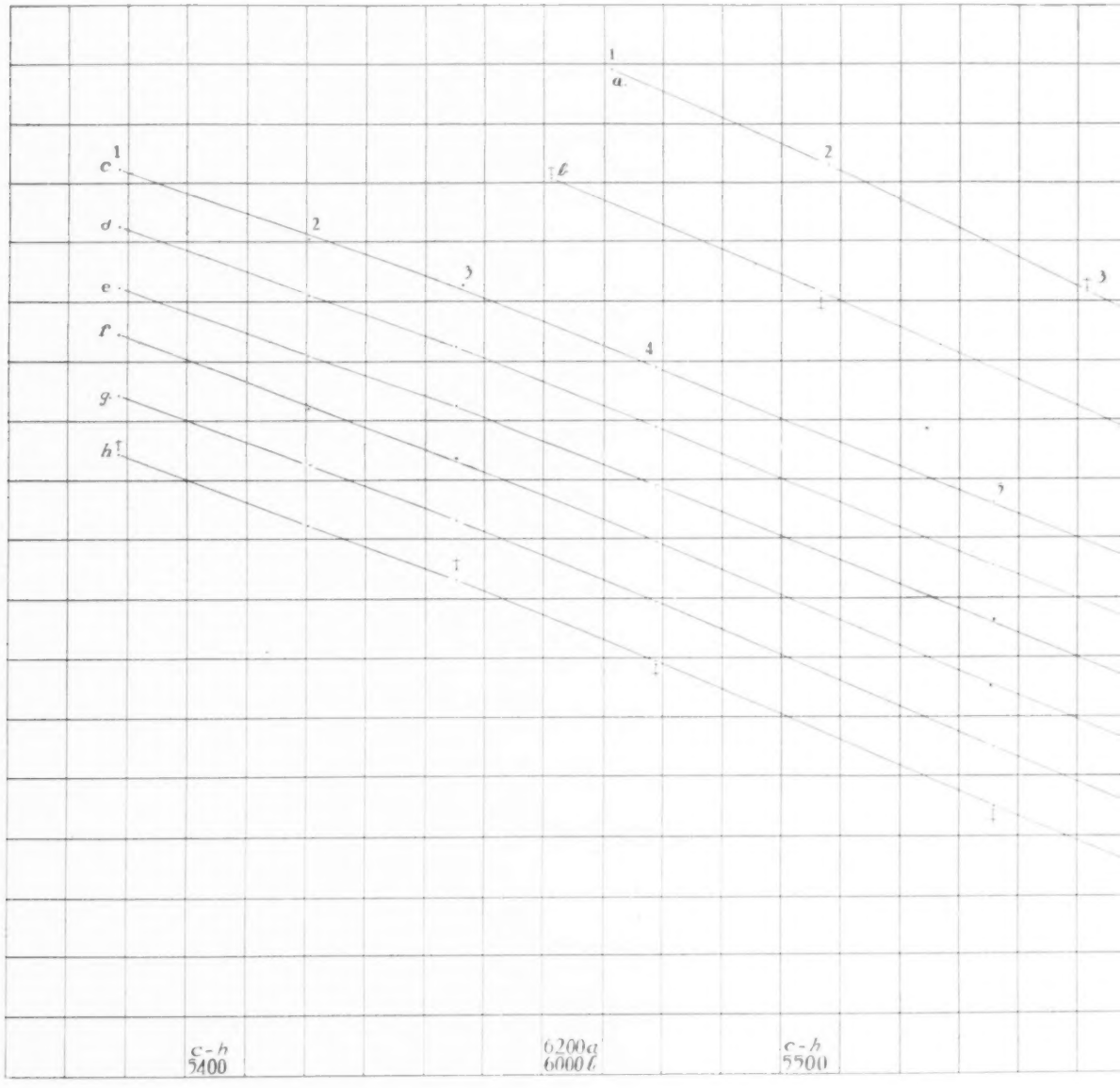
For this region international standards of the second order are available which belong to a single group when classified according to pressure-shift, and over the narrow region under investigation have the same pressure-shift per atmosphere. They have, therefore, the same relative wave-lengths under wide variations of pressure, and hence are suitable for comparing the wave-lengths of other lines measured both in Pasadena and on Mount Wilson. For convenience of reference these standards are assembled in Table II together with the data from Gale and Adams as to pressure-shift, Kayser's adjusted values, and their deviations from the international standards.

TABLE II

STANDARDS OF THE SECOND ORDER

International Standards	Group	Δ at 8 Atm.	Kayser's Adjusted	I.–K.
(5975.354)	*b*	0.054	5975.354	0.000
6027.059	*b*	0.062	6027.062	−0.003
6065.492	*b*	0.077	6065.489	+0.003
6137.701	*b*	0.078	6137.703	−0.002
6191.568	*b*	0.086	6191.568	0.000
6230.734	*b*	0.070	6230.734	0.000
6265.145	*b*	0.070	6265.145	0.000
6318.028	*b*	0.080	6318.031	−0.003
6335.341	*b*	0.074	6335.339	+0.002
6393.612	*b*	0.072	6393.612	0.000
6430.859	*b*	0.068	6430.848	+0.011
6494.993	*b*	0.065	6494.994	−0.001
Mean		0.071		
Mean shift per atmosphere		0.009		

The line at λ 5975 is not a standard of the second order, but a standard was needed near this wave-length, as it was desirable to include the two lines at λ 6003 and λ 6008 in the investigation. It was known from an inspection of the plates of Gale and Adams that these lines were enormously widened and displaced to the red, under a pressure of 9 atmospheres, and, in fact, Mr. Adams made a tentative measurement of the displacements of these two lines on one plate but did not consider the results of sufficient accuracy to


a
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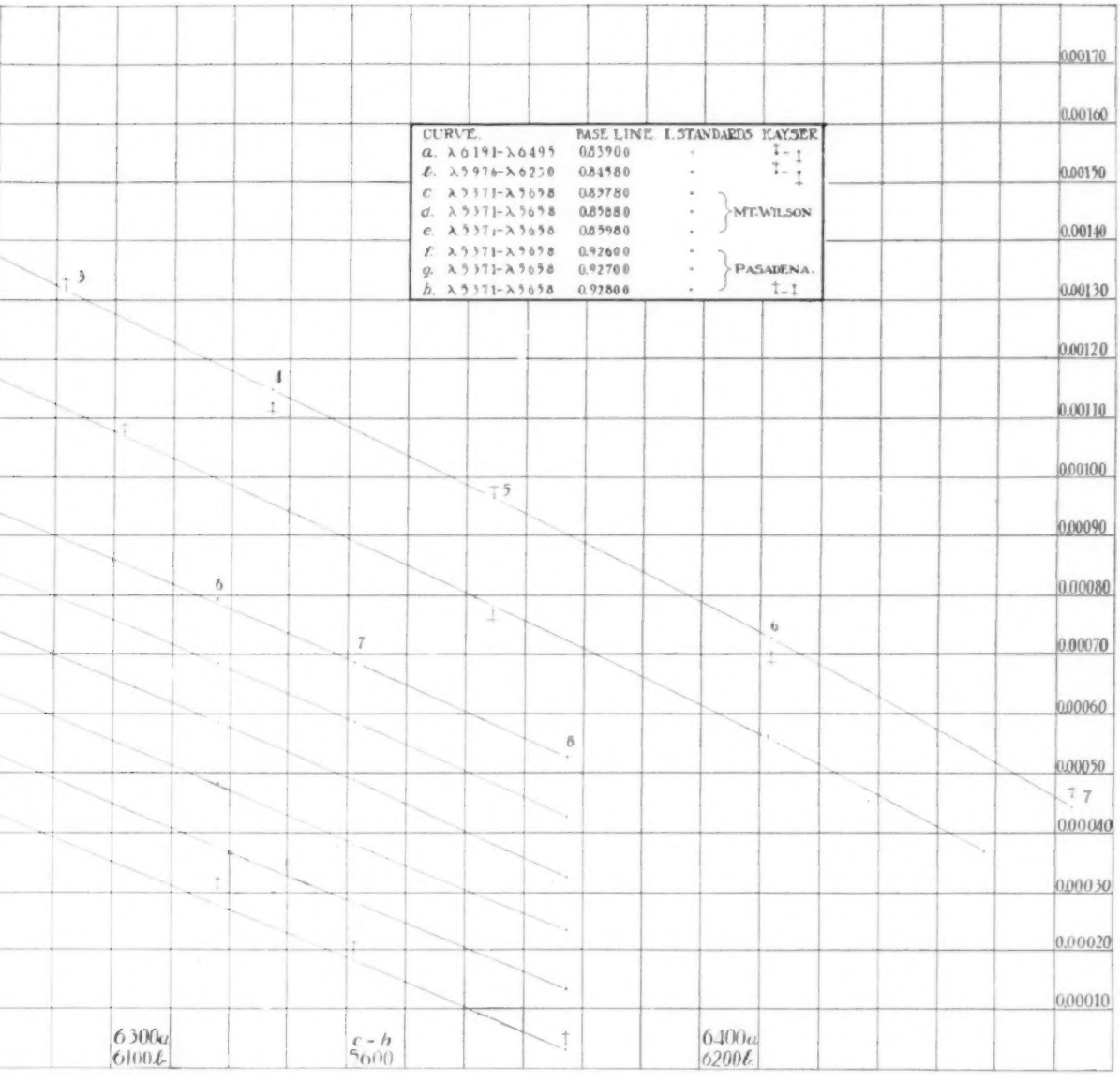

CURVE. BASE LINE. I. STANDARDS KAYSER
a. λ6191-λ6495 0.83900 ‡-‡
b. λ5976-λ6250 0.84580 ‡-‡
c. λ5371-λ5658 0.83780
d. λ5371-λ5658 0.85880 MT. WILSON
e. λ5371-λ5658 0.85980
f. λ5371-λ5658 0.92600
g. λ5371-λ5658 0.92700 PASADENA.
h. λ5371-λ5658 0.92800 ‡-‡
0.00170
0.00160
0.00150
0.00140
0.00130
0.00120
0.00110
0.00100
0.00090
0.00080
0.00070
0.00060
0.00050
0.00040
0.00030
0.00020
0.00010
6300a 6100ℓ
c - h 5600
6400a 6200ℓ

be included in their published list. They are given in our Table VI. The region is poor in lines and λ 5975 is not of good quality. It is, however, the only line available, and it belongs to the same group as the international standards, and therefore retains its relative wave-length at observing-stations at different elevations, an essential property of standards, as will appear in the course of this investigation. The wave-length given by Kayser for this line was adopted provisionally.

The inherent smoothness of the dispersion-curve for a plate makes it possible to test the consistency of a series of standards with a degree of accuracy which depends upon the length of the intervals between the standards and the precision of the measurements of the distances between the standards on the plate. This is of such great importance in the examination of standards that we give in Table III the complete data for such a curve extending over a range of about 300 Ångströms (between λ 6191 and λ 6494).

TABLE III

International Standards	Intervals	Distances	Factors	Mean λ	Kayser's Adjusted Values	Factors
6191.568					6191.568	
	39.166	45.7586 mm	0.85593	6211.151		0.85593
6230.734					6230.734	
	34.411	40.2802	0.85429	6247.940		0.85429
6265.145					6265.145	
	52.883	62.0582	0.85215	6291.586		0.85220
6318.028					6318.031	
	17.313	20.3567	0.85048	6326.684		0.85024
6335.341					6335.339	
	58.271	68.6640	0.84864	6364.476		0.84867
6393.612					6393.612	
	37.247	44.0138	0.84626	6412.236		0.84601
6430.859					6430.848	
	64.134	76.0395	0.84343	6462.926		0.84359
6494.993					6494.994	

The measurements were made on spectra of the second order taken on plates 43 cm long, inclined to the axis of the spectrograph to bring the plate tangent to the focal curve of the spectrographic lens of 30 feet (9 m) focus. In the sixth column are given the adjusted values according to Kayser, and in the seventh column the corresponding factors. Our curves were plotted with the reduction factors as ordinates on such a scale that 1/16 of an inch

(1.6 mm) represented one unit in the fourth place, and in interpolating we read the factor from the curve to the fifth significant figure. The wave-lengths were plotted as abscissae on such a scale that one of the above divisions represented one Ångström. The original curves from which Plate VIII was reduced were prepared in our draughting-room by Mr. Nichols; the data for the curves were given to him and he was requested to locate the points with great precision. Curve *a*, shown on this plate, corresponds to the data in the fourth and fifth columns of the table. A smooth curve was drawn, and, as is seen, the points fall upon the curve in a most satisfactory way. These data are the means of fourteen measurements and are practically free from accidental errors. Each measurement has given a similar curve but not quite so smooth as the composite curve. Upon the same plate are plotted the points given by using Kayser's adjusted values for the standards. The first three standards agree so that points 1 and 2 coincide. Point 3 is very slightly above, point 4 very much below, point 5 above, point 6 below, and point 7 above the smooth curve. That is, the points determined from Kayser's adjusted values fall in such a way that a smooth curve cannot be drawn upon which the points all lie. A glance at the curve shows at once how consistent with each other are the international standards, and how irregular are the adjusted values. The precision of the method is quite remarkable; a change of 0.002 Ångström in an interval distinctly displaces the corresponding point. The first interval changed by Kayser is the third—52.883 Ångströms—and though this long interval is lengthened only by 0.003 Ångström the ordinate of the point is increased five units in the fifth place. The fourth interval —17.313 Ångströms—is decreased by 0.005 Ångström which displaces the point downward twenty-four units in the fifth place. The fifth point is raised by increasing the interval from 58.271 to 58.273 Ångströms. The sixth point is an important one as it involves a line for which Kayser's adjusted value differs much from the international standard. In reference to this he says:[1]

> This [his] method of measurement has also yielded values for the standards themselves, which I regard as somewhat more accurate than the secondary

[1] *Astrophysical Journal*, **32**, 217, 1910.

international standards, inasmuch as they are adjusted. They are in general in good accord and only in a few cases is the difference large, in the case of λ 6430 reaching the amount of 0.012 Ångström. It remains to be investigated whether my value is more accurate than the other.

Referring to the curve, it will be seen that point 6 involving this line is greatly displaced; in fact, the point is lowered by 25 units in the fifth place; and point 7, also involving this line is raised 16 units in the fifth place. It will be noticed that if the wrong value is assigned to one wave-length of a series, the points in which this wave-length are involved are displaced one up and one down so that in such a case the value at fault can easily be ascertained. Judged by the results of this examination of these eight lines, the values for the international standards form a consistent series, no one of which probably contains an error exceeding 0.001 Ångström. The line λ 6430 is an excellent line for measurement, hard and sharp on all of our plates. The values by the three independent observers employing the interference method agree well, viz.:

Fabry-Buisson	Eversheim	Pfund
6430.859	6430.862	6430.855

It is difficult to understand how all of these measurements could be sufficiently in error for the wave-length to be such as Kayser gives it, namely 6430.848, a value not included within the range of the other three. It would tend to destroy faith in the precision of the interference method if such errors were found in the case of lines so excellent for measurement as λ 6430. The method which we illustrate of testing the adjustment of standards is capable of detecting an error of 0.002 Ångström in the case of standards 50 Ångströms apart, and depends only upon the accuracy of the measurements of their distances apart on the photographic plate, values which can be determined to a very high degree of precision. It is essential that a sufficient number of standards be found upon one plate, and with high dispersion this condition requires a plate of great length. These conditions will be well fulfilled by the new 75-foot spectrograph on Mount Wilson, in which a plate 40 inches (100 cm) long can be used. A series of overlapping plates is neces-

sary in order that the lines under investigation may not be too near the end of the series under consideration. The curve *b* on Plate VIII is drawn from data similar to that given in Table III, and is based on 28 measurements of plates covering the region λ 5975–λ 6265, thus overlapping the first region so that points 5 and 6 of this curve are points 1 and 2 on curve *a*. The international and Kayser's adjusted values are:

International..	(5975.354)	6027.059	6065.492	6137.701	6191.568	6230.734	6265.145
Kayser.....	.354	.062	.489	.703	.568	.734	.145
Int.−Kayser..	.000	−.003	+.003	−.002	.000	.000	.000

As in the other region, the points determined from the international values fall on the curve in a very satisfactory way, while the four points involving the three adjusted values fall alternately above and below. The line λ 5975, as before mentioned, is not an international standard, but the value assigned to it by Kayser appears to fit into the series of international standards. It belongs to the same group when classified according to pressure-shift. It is, however, a line difficult to measure with accuracy and hence not suitable for a standard of the second order. Judged by this curve the relative wave-lengths of the international standards λ 6027–λ 6137 are free from errors as great as 0.001 Ångström.

REGION λ 5371–λ 5658

This region is considered by itself because of two difficulties that arise. The first is due to the fact that the international standards involved belong to two groups when classified according to pressure-shift, and that in these groups the relative wave-lengths change a measurable amount with a decrease in pressure of one-fifth of an atmosphere, the change between Pasadena and the observatory on Mount Wilson. The second difficulty is that some adjustments are indicated in the international standards. The standards and related data are given in Table IV.

The first section of the table contains lines of group *a* (Gale and Adams), the flame lines, for which the displacements at 8 atmospheres were very accurately determined by them. The lines

in the second section of the table were provisionally classified by them in group *d*, the lines showing great widening and displacement to the red under pressure. The precision of measurement of the displacements of these lines at 8 atmospheres is low, as they have said. The relative change in wave-length between the two sections of the table is 0.003 Ångström for a pressure-change of one-fifth of an atmosphere. After our previous experience in the red region we expected to detect this change in wave-length upon the plates taken upon Mount Wilson. It ought to manifest itself by the point

TABLE IV

STANDARDS OF THE SECOND ORDER

International Standards	Group	Δ at 8 Atmospheres	Kayser	I.–K.
5371.495	*a*	0.029	5371.490	+0.005
5405.780	*a*	0.027	5405.778	+0.002
5434.527	*a*	0.027	5434.527	0.000
5455.614	*a*	0.029	5455.616	−0.002
5497.522	*a*	0.030	5497.521	+0.001
5506.784	*a*	0.031	5506.783	+0.001
Mean Δ		0.029		
Mean Δ per atmosphere		0.0036		
5569.633	sub *d*	0.14	5569.630	+0.003
5586.772	sub *d*	0.12	5586.774	−0.002
5615.661	sub *d*	0.13	5615.667	−0.006
5658.836	sub *d*	0.15	5658.846	−0.010
Mean Δ		0.14		
Mean Δ per atmosphere		0.017		

determined by λ 5506 and λ 5569, the terminal and initial lines of the two sets of standards, being slightly displaced upward on the dispersion-curve. We show the results in curve *c* in Plate VIII. The data for the curve are the means of 20 measurements and should be practically free from accidental errors. The mean of the two standards λ 5497 and λ 5506 was used in plotting the curve in order to weight the intervals between standards more equally. In the case of an interval so short as that between these two lines, an error of a few ten-thousandths of an Ångström in their wave-length, or a few ten-thousandths of a millimeter in the measured distance between the lines, displaces the point very markedly.

Point 5 is the one determined from λ 5506 and λ 5569. It falls, however, very near the curve, while point 6 is distinctly above the curve. This was somewhat surprising, and at first a little disconcerting. The points 2 and 3 are plainly displaced, indicating a small error in the wave-length of λ 5434, the only line concerned with both points. Later when the plates taken in the laboratory in Pasadena were reduced and the dispersion-curves for this region were plotted it at once became evident where the difficulty lay. Curve *f*, Plate VIII, is based upon 20 measurements made on plates taken in Pasadena. Points 2 and 3 are displaced as on the curve for the Mount Wilson plates, confirming the indication of a probable

TABLE V

PROPOSED ADJUSTMENTS

INTERNATIONAL STANDARDS	PASADENA	MOUNT WILSON	
		A	B
5371.495	5371.495	5371.495	5371.495
5405.780	5405.780	5405.780	5405.780
5434.527	5434.529	5434.529	5434.529
5455.614	5455.614	5455.614	5455.614
5497.522	5497.522	5497.522	5497.522
5506.784	5506.784	5506.784	5506.784
5569.633	5569.634	5569.631	5569.634
5586.772	5586.772	5586.769	5586.772
5615.661	5615.661	5615.658	5615.661
5658.836	5658.836	5658.833	5658.836

error in λ 5434. It also shows displacements of points 5 and 6 from the mean curve, thus indicating an error in λ 5569. The wave-lengths of the lines λ 5434 and λ 5569, in terms of the other standards, can be obtained by interpolation from these curves. The wave-lengths so determined differ by 0.002 and 0.001 Ångström respectively from the international means for these lines. The second, third, and fourth columns of Table V are adjusted to these values.

Curves are given on Plate VIII corresponding to the four columns of Table V. Those based upon the first column are the curves *c* and *f*, showing the relative wave-lengths of the unadjusted stand-

ards for Mount Wilson and Pasadena, respectively. Curve *g* is based upon the adjusted wave-lengths given in the second column as determined from the measurement of the standards on the Pasadena plates. It now has a smooth run and so also has curve *d*, which corresponds to the third column in which the suggested changes have been made on λ 5434 and λ 5569. The wave-lengths of the second group of standards have been corrected for their displacement of 0.003 Ångström, on account of the decreased atmospheric pressure on the mountain, relative to the flame lines of the first section of the table. This change having been allowed for, they ought to form a consistent series; as is seen, the points determined from the mountain measurements now fall upon the curve. Then curve *e* was drawn, using the wave-lengths in the fourth column, in which the suggested changes in the Pasadena values have been made so that they form a consistent series, but no consideration is taken of the relative change in wave-length of the two groups of standards under the pressure-change of one-fifth of an atmosphere that occurs in passing from the altitude of Pasadena to the observatory on Mount Wilson. With these adjustments in the low-altitude values, points 2 and 3 fall on the curve, and point 5, depending upon the terminal and initial lines of the two groups of standards, is displaced slightly upward. It was expected that this would be the case when the mountain plates for this region were first reduced and the point was found to fall on the curve. It appears from the observational data here given that the international values for the wave-lengths of λ 5434 and λ 5569 are slightly in error, and that under the decreased pressure on Mount Wilson the wave-length of λ 5569 was shortened so that its actual wave-length differed only by 0.002 Ångström from that given in the table of the international standards. At the altitude of Mount Wilson it formed a part of a practically consistent series with the flame lines, and therefore no great irregularity was shown in the curve at the point where the two groups joined. The uncertainty regarding point 5 in the first reduction of the mountain plates, using the international values, was caused by the long interval—63 Ångströms—between the terminal and initial lines of the two groups of standards, by the partial compensation by the displacement due

to decrease of pressure, and especially by the relatively large displacement of point 6 in the same direction due to the small error in λ 5569 effective over the short interval—17 Ångströms—between it and λ 5586. The manner in which these results confirm one another, however, furnishes strong grounds for believing that they correspond to the facts of the case, and illustrates the precision of the method. The corrections suggested are within the range covered by the international determinations of the individual standards, as the comparison shows.

Fabry and Buisson	Eversheim	Pfund	Suggested	Kayser
5434.530	.524	.528	.529	.527
5569.632	.636	.631	.634	.630

Of the ten standards in this region Kayser suggests adjustments in nine. In case successive lines are given algebraically equal increments, the interval is not changed, but the intervals preceding and following such a series of adjustments are altered and the positions of the corresponding points of the dispersion-curve therefore displaced. The points for the dispersion-curve calculated from Kayser's adjusted wave-lengths in the fourth column of Table IV are shown on curve *h*, Plate VIII. The smooth curve is the same as *g*. Point 2 falls upon the curve. The standards inclosing this point differ by equal increments from those upon which curve *g* is based, so that the interval remains unaltered; the preceding and following points, however, are displaced; 1 and 3 by 8 and 18 units respectively in the fifth place; 4 and 5 in the opposite direction by 5 units in the fifth place. Especially to be considered are points 6, 7, and 8, as these depend upon wave-lengths for which Kayser's suggested adjustments are −0.003, +0.002, +0.006, and +0.010 Ångström. The interval to which point 6, greatly displaced, corresponds is a short one—17 Ångströms—so that decreasing one standard by 0.003 Ångström and increasing the other by 0.002 Ångström produced a change in the factor of 32 units in the fifth place. The position of point 8 depends upon the relative wave-lengths of λ 5615 and λ 5658. Professor Kayser suggests an increase of 0.006 Ångström for the first and an increase of 0.010

Ångström for the last. The point is thereby raised by 9 units in the fifth place, but if the value for λ 5615 had not been increased, point 8 would have been raised 13 units more; that the suggested changes in the preceding wave-lengths were not justified is shown by the positions of points 6 and 7. The evidence, therefore, from this part of the curve is opposed to these changes in the international standards. The adjustments suggested by Professor Kayser indicated that large and systematic errors were involved in the determinations of the international standards by the interference method, as the suggested wave-lengths for these lines are not embraced within the values found by the three independent observers.

Fabry and Buisson	Eversheim	Pfund	Kayser
5615.658	.662	.663	.667
5658.835	.838	.835	.846

From the evidence furnished by the dispersion-curves it appears that none of the international standards in the region λ 5371–λ 5658 is affected with a relative error as great as 0.002 Ångström except λ 5434, and that for other lines the error probably does not amount to 0.001 Ångström except for λ 5569. If any change in the wave-length of λ 5658 is indicated by the dispersion-curve, it is a small decrease rather than a large increase; the only point, however, involving this line is the terminal one, and owing to its position the amount of the error, in case there is a small one, cannot be determined accurately; yet the evidence seems decisive against the large positive correction suggested by Professor Kayser.

The displacements of a few units in the fifth place of the factors are well within the limits of error that occur in reading the factors from the curves. We have usually checked each other when taking the factors from the curves in the interpolations for the standards of the third order, and we rarely differ by 2 units in the fifth place. The scale is greatly reduced in the reproduction shown on Plate VIII, but the larger displacements are easily seen. With the dispersion of the plates taken in the second order an error of 0.002 Ångström in the wave-length of the standard, when referred to the neighboring standards at a distance of 50 Ångströms, would displace the

points involving the line in error by 3 units in the fifth place, alternately up and down, and by proportionately greater amounts in the case of more closely spaced standards. The delicacy of the method appeared in adjusting the corrections to λ 5434 and λ 5569. These were found to be slightly in excess of 0.002 and 0.001 Ångström respectively; in using the values 0.003 and 0.002 Ångström it was seen that the lines were overcorrected, while in the case of the values suggested they are slightly undercorrected.

In obtaining the international standards of the second order, it was hoped that the standards would be accurate to 0.001 Ångström in the visible spectrum, and it appears from our examination of the region λ 5371–λ 6494 that practically this hope has been realized, as we have not found errors to exceed that limit except in the case of one line. With secondary standards whose wave-lengths are known to such exactness it will be easily possible to reach a like precision for good lines in the case of the tertiary standards.

WAVE-LENGTH DETERMINATION

REGION λ 5371–λ 6494

In arranging the results of our measurements we found that the lines showing displacements to the red under pressure fall into the groups proposed by Gale and Adams in an important paper on the spectra of iron and titanium under pressure.[1]

Group a.—This involves the well-known flame lines,[2] i.e., the lines relatively strengthened in low-temperature sources, such as the flame of the arc, the low-current arc, and the electric furnace.[3] The lines of this group in the yellow-green show small but definite pressure displacements, the mean being 0.0036 Ångström per atmosphere in the arc.[4]

[1] *Contributions from Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

[2] De Watteville, *Philosophical Transactions*, A **204**, 139, 1904.

[3] Hale, Adams, and Gale, *Contributions from the Mount Wilson Solar Observatory*, No. 11; *Astrophysical Journal*, **24**, 185, 1906; Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 86, 1909; King, *Contributions from the Mount Wilson Solar Observatory*, No. 53; *Astrophysical Journal*, **34**, 37, 1911; King, *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[4] Gale and Adams, *op. cit.*

Group b.—To this group many lines belong; in fact all lines of moderate displacement under pressure are assigned to it for the present. These are bright and symmetrically widened under pressure, and show a mean pressure displacement of 0.009 Ångström per atmosphere for the lines in the region λ 5975–λ 6678 according to Gale and Adams.

Group c.—Group *c* has no representatives in this region.

Group d.—This contains lines showing immense displacements to the red under pressure. The lines are bright and greatly widened to the red under pressure. We have been led to divide group *d*, calling the separate portion sub-group *d*. The lines are similar in behavior under pressure, but show much smaller displacements.

Group e.—To the groups suggested by Gale and Adams we have added a group consisting of lines that remain bright and are greatly displaced to the violet and unsymmetrically widened to the violet under pressure.

DISCUSSION

LINES GREATLY DISPLACED TO THE RED

In Table VI the Pasadena and Mount Wilson values for the wave-lengths are given in the second and third columns respectively. The sixth column contains the differences Pasadena minus Mountain; the ninth and tenth columns show the differences Kayser minus Pasadena or minus mean, and Rowland minus Pasadena or minus mean. In the last column are given the displacements for 8 atmospheres according to Gale and Adams, in the case of lines for which they have given the data. For the other lines we give the results from inspection of their plates. When the lines are so diffuse under pressure that no measurements of value can be made we have said "unmeasurable," though the displacement of the maximum is easy to see. In some cases it has not been possible to fix the type by inspection, but the differences Pasadena—Mount Wilson, and Rowland—Pasadena have shown the group to which the line belongs.

The mean difference, Pasadena—Mount Wilson, for the lines of group *d* in the yellow-green is 0.0075 Ångström. This corresponds to a change in pressure of one-fifth of an atmosphere, or a

TABLE VI

LINES GREATLY DISPLACED AND UNSYMMETRICALLY WIDENED UNDER PRESSURE

Not Recommended for Standards

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.–MT.	WIDTHS IN ÅNGSTRÖMS		KAYSER–PASADENA	ROWLAND–PASADENA	UNDER 9 ATMOSPHERES
	Pasadena	Mount Wilson	P.	Mt.		Pasadena	Mt.			
Iron *d*, green	5393.188(20)	.182(21)	5	4	+0.006	0.100	0.105	−0.002	0.187	Δ=0.14 Å
	5476.587(33)	.578(10)	4	3	+0.009	0.108	0.113	−0.006	0.191	Δ=0.11
					+0.0075	0.104	0.109		0.189	
								K.–Mean	R.–Mean	
	5569.634(s)							−0.004	0.214	Δ=0.14 Å
	5572.859(9)	.858(10)	4	3	+0.001	0.120	0.128	−0.001	0.216	Δ=0.14
	5576.109(17)	.107(15)	6	5	+0.002	0.086	0.090	−0.006	0.212	Δ=0.16
	5586.772(s)							+0.002	0.219	Δ=0.12
Iron sub *d*	5602.958(5)	.964(11)	4	4	−0.006	0.100	0.100	+0.001	0.225	Δ=0.15
	5615.661(s)							+0.006	0.216	Δ=0.13
	5624.559(19)	.560(12)	3	2	−0.001	0.125	0.126	+0.003	0.210	Δ=0.16
	5638.275(11)	.280(4)	4	3	−0.005	0.078	0.108	+0.012	0.211	Δ=0.15
	5658.836(s)							+0.010	0.216	Δ=0.15
					−0.002	0.102	0.110		0.215	
								K.–P.	R.–P.	
	5976.816(14)	.804(19)	5	4	+0.012	0.093	0.105	−0.016	0.191	Diffuse. Max. to red. Unmeas.
	5983.712(18)	.705(28)	6	2	+0.007	0.119	0.135	−0.003	0.196	5984 displ. to v. overlaps
Iron *d*, red	6003.051(11)	.038(26)	6	6	+0.013	0.121	0.125	−0.022	0.188	Δ=0.38 Å
	6008.595(11)	.581(25)	6	7	+0.014	0.124	0.125	−0.015	0.190	Δ=0.40
	6147.844(6)	.836(8)	4	3	+0.008	0.064	0.102	0.000	0.196	Indeterm. Normal arc weak
	6151.636(4)	.628(4)	4	4	+0.008	0.097	0.107	−0.003	0.198	Indeterm. Normal arc weak

Iron *d*, red —*Cont'd*	6180.225(7)	.216(10)	5	6	+0.009	0.084	0.120	+0.005	0.195	Indeterm. Normal arc weak
	6232.669(11)	.662(24)	4	3	+0.007	0.089	0.101	0.000	0.187	Diffuse. Max. to red. Unmeas.
	6246.350(12)	.337(36)	4	4	+0.013	0.123	0.135	−0.005	0.185	Δ=0.28 Å
	6301.531(16)	.518(18)	4	4	+0.013	0.093	0.109	−0.003	0.187	Δ=0.25
	6302.520(11)	.511(17)	4	3	+0.009	0.083	0.084	+0.002	0.189	Diffuse. Max. to red. Unmeas.
	6336.851(16)	.843(17)	4	3	+0.008	0.084	0.098	−0.001	0.197	Δ=0.26 Å
	6400.026(6)	.020(14)	4	2	+0.006	0.123	0.137	+0.010	0.191	Δ=0.24
	6408.044(13)	.038(8)	3	3	+0.006	0.072	0.096	+0.001	0.189	Diffuse. Max. to red. Unmeas.
	6411.678(16)	.668(14)	3	3	+0.010	0.108	0.137	−0.005	0.187	Δ=0.23 Å
					+0.0095	0.098	0.114	+0.018 −0.073	0.191	
Mn	6013.528(20)	.516(5)	4	4	+0.012	0.108	0.109	−0.012	0.187	Diffuse. Max. to red. Unmeas.
	6016.668(17)	.663(13)	6	8	+0.005	0.118	0.123	−0.002	0.193	Diffuse. Max. to red. Unmeas.
	6021.830(13)	.824(9)	5	3	+0.006	0.133	0.137	−0.009	0.186	Diffuse. Max. to red. Unmeas.
					+0.008	0.120	0.123	−0.023	0.189	
Iron *e*	5383.353(7)	.371(8)	9	4	−0.018	0.141	0.147	+0.007	0.225	Diffuse. Max. to v. Unmeas.
	5410.890(13)	.911(8)	6	2	−0.021	0.096	0.108	+0.012	0.234	Diffuse. Max. to v. Unmeas.
	5415.175(10)	.195(13)	6	3	−0.020	0.118	0.122	+0.009	0.241	Diffuse. Max. to v. Unmeas.
	5424.038(11)	.060(12)	6	5	−0.022	0.136	0.138	+0.012	0.252	Diffuse. Max. to v. Unmeas.
	5543.173(7)	.185(5)	6	4	−0.012	0.096	0.106	+0.011	0.241	Indeterm.
	5554.872(14)	.887(3)	6	3	−0.015	0.090	0.109	+0.006	0.250	Diffuse. Max. to v. Unmeas.

TABLE VI—*Continued*

Group	International Wave-Lengths		Mean Deviation		P.–Mt.	Widths in Ångströms		Kayser–Pasadena	Rowland–Pasadena	Under 9 Atmospheres
	Pasadena	Mount Wilson	P.	Mt.		Pasadena	Mt.			
Iron *e* —*Cont'd*	5565.688(17)	.702(4)	7	4	−0.014	0.111	0.112	−0.003	0.243	5563 displ. to red overlaps
	5598.279(10)	.303(4)	8	1	−0.024	0.105	0.125	+0.013	0.245	No material
	5984.796(15)	.807(26)	5	7	−0.011	0.124	0.139	−0.001	0.244	5983 displ. to red overlaps
	5987.050(17)	.058(28)	3	4	−0.008	0.120	0.135	−0.005	0.240	Diffuse. Max. to v. Unmeas.
	6007.961(9)	.969(17)	5	5	−0.008	0.125	0.150	−0.008	0.225	Indeterm. Normal arc weak.
	6024.051(16)	.059(29)	5	4	−0.008	0.188	0.199	−0.004	0.230	Diffuse. Max. to v. Unmeas.
	6042.083(11)	.093(20)	6	4	−0.010	0.088	0.109	+0.009	0.232	Diffuse. Max. to v. Unmeas.
	6055.983(11)	.995(22)	6	6	−0.012	0.107	0.133	+0.009	0.244	Diffuse. Max. to v. Unmeas.
	6078.470(14)	.482(21)	8	5	−0.012	0.135	0.136	+0.006	0.240	Diffuse. Max. to v. Unmeas.
					−0.014	0.119	0.131	+0.094 −0.021	0.239	

displacement per atmosphere of 0.0375 Ångström relative to the standards, on the assumption that the differences, Pasadena—Mount Wilson, are attributable entirely to pressure displacement. The standards are displaced to the red 0.0036 Ångström per atmosphere. The absolute displacement is therefore 0.041 Ångström per atmosphere, an amount in excess of the values found by Gale and Adams for these lines, but they say their measurements were little more in some cases than "an estimate of the maximum within the broad band." The mean difference, Pasadena—Mount Wilson, for the lines of group *d* in the red is 0.0095 Ångström; this becomes 0.048 Ångström per atmosphere relative to the standards of group *b*, which show displacements to the red of 0.009 Ångström per atmosphere. The absolute displacement to the red is then 0.057 Ångström per atmosphere, an amount again in excess of the value found by Gale and Adams. On the whole, however, the degree of agreement is surprising rather than otherwise. Assuming that the displacement varies as the cube of the wave-length, the relation found by Gale and Adams for the lines of iron belonging to a common group, the agreement between the observed and calculated values for group *d*, yellow-green, based upon the value 0.057 Ångström for group *d*, red, is within the limits of error, viz., group *d* (yellow-green), observed 0.041 Ångström, calculated 0.038 Ångström. Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine *in vacuo* and under normal pressure the behavior of an extended list of lines belonging to groups *d* and *e*.

Sub-Group d.—The mean displacement per atmosphere for the lines of this group, found by Gale and Adams, is 0.019 Ångström to the red. Our results are in complete agreement with this, as we found that the intervals between the international standards of this group and the standards of group *a*, the flame lines, remained constant when the relative displacements for a decrease of pressure of one-fifth of an atmosphere were taken into account. As a basis, displacements of 0.0036 Ångström per atmosphere for the flame-line standards and of 0.019 Ångström per atmosphere for

TABLE VII

LINES MODERATELY DISPLACED AND SYMMETRICALLY WIDENED UNDER PRESSURE
Recommended for Standards

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.–MT.	MEAN	KAYSER–MEAN	ROWLAND–MEAN	UNDER PRESSURE OF 9 ATM.
	Pasadena	Mount Wilson	P.	Mt.					
a.....	5371.495(s)	.....	..	..	.....	.....	−0.005	complex	Δ=0.029 Å
a.....	5405.780(s)	.....	..	..	.....	.....	−0.002	0.209	Δ=0.027
a.....	5429.702(13)	.702(20)	1	2	0.000	5429.702	−0.001	0.209	Δ=0.029
a.....	5434.529(s)	.....	..	..	.....	.....	−0.002	0.211	Δ=0.027
a.....	5446.919(33)	.920(20)	3	3	−0.001	5446.919	−0.001	0.211	Δ=0.031
a.....	5455.614(s)	.....	..	..	.....	.....	+0.002	0.220	Δ=0.029
a.....	5497.522(s)	.....	..	..	.....	.....	−0.001	0.213	Δ=0.030
a.....	5501.469(33)	.471(15)	2	2	−0.002	5501.470	−0.001	0.213	Δ=0.030
a.....	5506.784(s)	.....	..	..	.....	.....	−0.001	0.216	Δ=0.031
a.....	5535.419(12)	.419(12)	3	4	0.000	5535.419	+0.001	0.225	Δ=0.034
?.....	5543.944(1)	.944(12)	..	..	0.000	5543.944	0.000	0.213	
?.....	5563.613(15)	.612(5)	4	4	+0.001	5563.613	−0.005	0.211	
							+0.003	0.214	
							−0.019		
b.....	5975.354	.....	..	..	.....	.....	0.000	0.221	Δ=0.054 Å
b.....	6027.059(s)	.....	..	..	.....	.....	+0.003	0.215	Δ=0.062
b.....	6065.492(s)	.....	..	..	.....	.....	−0.003	0.217	Δ=0.077
?.....	6127.913(15)	.912(16)	5	3	+0.001	6127.912	+0.015	0.212	.....
b.....	6136.626(22)	.624(21)	1	2	+0.002	6136.625	+0.003	0.204	Δ=0.082
?.....	6136.998(14)	7.000(12)	5	3	−0.002	6136.999	+0.013	0.211	.....
b.....	6137.701(s)	.....	..	..	.....	.....	+0.002	0.214	Δ=0.078
b.....	6157.731(18)	.731(13)	5	2	0.000	6157.731	+0.009	0.214	Δ=0.041
?.....	6165.367(18)	.366(18)	5	3	+0.001	6165.366	+0.021	0.211	.....
b.....	6173.341(21)	.343(17)	7	2	−0.002	6173.342	+0.008	0.211	Δ=0.067
b.....	6191.568(s)	.....	..	..	.....	.....	0.000	0.211	Δ=0.086

b.....	6200.322(9)	.322(21)	3	3	0.000	6200.322	+0.008	0.205	Δ=0.079 Å
b.....	6213.437(12)	.439(31)	4	2	−0.002	6213.438	0.000	0.206	Δ=0.072
?.....	6215.150(9)	.152(19)	5	4	−0.002	6215.151	+0.002	0.209	.....
b.....	6219.287(17)	.290(34)	5	2	−0.003	6219.289	0.000	0.205	Δ=0.073
b.....	6230.734(s)	.....	..	..	.....	.....	0.000	0.209	Δ=0.070
b.....	6252.569(17)	.567(37)	3	2	+0.002	6252.568	−0.001	0.205	Δ=0.077
b.....	6254.267(17)	.268(37)	2	2	−0.001	6254.268	+0.001	complex	Δ=0.064
b.....	6265.145(s)	.....	..	..	.....	.....	0.000	0.203	Δ=0.070
b.....	6297.800(24)	.802(40)	2	2	−0.002	6297.801	+0.003	0.206	Δ=0.068
?.....	6315.310(14)	.313(22)	3	3	−0.003	6315.312	+0.011	0.205	.....
b.....	6318.028(s)	.....	..	..	.....	.....	+0.003	0.211	Δ=0.080
?.....	6322.693(21)	.695(34)	2	2	−0.002	6322.694	+0.004	0.213	.....
b.....	6335.341(s)	.....	..	..	.....	.....	−0.002	0.213	Δ=0.074
?.....	6344.159(2)	.159(6)	0	4	0.000	6344.159	+0.005	0.212	.....
?.....	6355.040(7)	.039(9)	2	2	+0.001	6355.039	+0.006	0.207	.....
b.....	6393.612(s)	.....	..	..	.....	.....	0.000	0.208	Δ=0.072
b.....	6421.363(16)	.362(14)	2	2	+0.001	6421.363	−0.009	0.207	Δ=0.068
b.....	6430.859(s)	.....	..	..	.....	.....	−0.011	0.207	Δ=0.068
b.....	6494.993(s)	.....	..	..	.....	.....	+0.001	0.220	Δ=0.065
							+0.118 −0.026	0.210	

the lines of sub-group *d* were employed. In other words, when the international standards of those two groups formed a consistent system in Pasadena they remained consistent on Mount Wilson when the displacements of the lines of sub-group *d* relative to the lines of group *a* were allowed for, but otherwise not. That the lines of sub-group *d* do not belong to group *d* is again clearly evidenced from the values of the differences, Rowland—Pasadena, of which more will be said later.

Sub-group *d*, as far as now known, consists of 9 lines in the yellow-green, 4 of which are among the international standards of the second order. The other 5 were measured on the Pasadena and Mount Wilson plates with a mean difference between the two stations of 0.002 Ångström. If the lines belong to the same group as the standards used, in this case those of sub-group *d*, the two measurements should agree. Considering the extremely poor quality of the standards and of the measured lines, the agreement may be regarded as satisfactory, their displacements under pressure agreeing within the limits of error with the displacements of the standards, a result already found by Gale and Adams.

In dealing with standards of this quality, the question of their availability for such an exacting purpose at once arises. These lines are greatly displaced and unsymmetrically widened under pressure. If it were a case of displacement only, the lines still remaining symmetrical, adjustments could be made for such changes of pressure as occur between sea-level and the mountain observatories, and the lines would still be usable for standards; but they are in practice usually unsymmetrical, the amount of dissymmetry depending upon the pressure and the quantity of vapor in the source, and, in general, two observers will not agree upon individual lines. This has been the case with the authors of this paper, but the means derived from the measures made upon a large number of lines show a good degree of agreement. To be of service in a series of standards in which errors are expected not to exceed 0.002 Ångström, the quality of a line should be such that a reasonable number of measurements will give that degree of precision. We show in Table IX the results of our individual measures of the distances in millimeters between the lines on 10 exposures

TABLE VIII

LINES OF QUESTIONABLE UTILITY AS STANDARDS

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.–MT.	MEAN	K.–MEAN	REMARKS
	Pasadena	Mount Wilson	P.	Mt.				
	5379.588(4)	.587(3)	1	2	+0.001	5379.588	−0.005	Material insuff. Quality poor
	5400.507(6)	.504(4)	3	4	+0.003	5400.506	+0.004	Material insuff. Quality poor
	5445.039(8)	.037(12)	5	6	+0.002	5445.038	−0.001	From best plates. Very faint
?.....	5462.958(4)	.956(5)	3	2	+0.002	5462.957	+0.007	From plates showing best separation. Lines diffuse. Merged under pressure. Like λ 6102–6103
?.....	5463.268(9)	.268(4)	2	2	0.000	5463.268	+0.004	
	5473.917(28)	.916(6)	3	5	+0.001	5473.917	−0.007	Measures difficult and uncertain
a.....	5476.295(33)	.298(3)	3	4	−0.003	5476.296	−0.002	Near diffuse line λ 5476.587. Sep. difficult
b?....	6020.178(5)	.176(21)	7	2	+0.002	6020.177	−0.010	Fe line distant 0.169 Å
	6089.571(14)	.568(11)	5	3	+0.003	6089.570	+0.011	Too faint on our plates for good measures
?.....	6102.179(13)	.181(21)	3	3	−0.002	6102.180	+0.002	Diffuse. Merges with 6103 under pressure. Like λ 5462–5463
?.....	6103.183(2)	.181(2)	8	2	+0.002	6103.182	+0.010	Fe line distant 0.114 Å
b.....	6256.	.375(35)	..	2	.....	6256.375	+0.002	Blend with nickel on valley plates
	6280.623(3)	.628(28)	4	4	−0.005	6280.626	+0.005	Insuff. valley material. Very faint
	6462.740(8)	.737(10)	8	2	+0.003	6462.738	−0.004	Insuff. material for difficult line

upon a plate in the third order, covering this region, with a scale 1 mm = 0.58 Ångström. We include two lines of group *a* for a comparison between the measures upon good and poor lines. The mean deviations are given in the last line. The abbreviations S. and W. refer to St. John and Ware.

TABLE IX

QUALITY OF SECONDARY STANDARDS λ 5569–λ 5658

GROUP *a*		SUB-GROUP *d*							
5497–5506		5506–5569		5569–5586		5586–5615		5615–5658	
S.	W.	S.	W.	S.	W.	S.	W.	S.	W.
15.918	.920	108.428	.427	29.715	.710	50.175	.174	75.283	.277
.917	.918	.430	.430	.696	.698	.178	.181	.287	.286
.916	.920	.419	.421	.708	.708	.177	.176	.281	.286
.918	.923	.432	.431	.692	.698	.173	.176	.289	.284
.921	.917	.423	.425	.692	.695	.178	.180	.302	.301
.920	.918	.431	.432	.699	.690	.180	.186	.286	.287
.917	.917	.431	.428	.682	.675	.188	.184	.294	.298
.920	.916	.424	.420	.700	.706	.178	.177	.286	.290
.920	.917	.439	.430	.688	.703	.175	.178	.288	.286
.919	.920	.440	.424	.694	.697	.175	.177	.299	.295
15.9186	.9186	108.430	.427	29.697	.698	50.178	.179	75.289	.289
0.0014	.0017	.0049	.0034	.0072	.0070	.0027	.0031	.0051	.0056

The agreement between our measures for the group *a* interval is complete; and if we had been obliged to rely upon one or two exposures, the divergence from the general mean would not have made a serious error. In the case of intervals involving the lines of sub-group *d*, however, our measures for the same exposures vary greatly and capriciously, but the general means are in fair agreement. It is evident that very divergent results would be obtained by using single exposures, and even a fair degree of accuracy can be had only by the use of an inconveniently large number of plates, a sufficient quantity of which in many cases would not be available.

Lines of the character of sub-group *d* will present different appearances upon spectra taken with concave and plane gratings. The astigmatism of the concave grating greatly lengthens the lines and tends to obscure the dissymmetry by distributing it along the length of the line. This would be a source of error, particularly if

the ends of the lines were cut off as in the case of a close-lying comparison spectrum. When the image of the middle of the arc is held stationary upon the slit of a plane-grating spectrograph, the dissymmetry of such a line becomes apparent; but when the image plays over the slit as in the case of an unsteady arc, the same line upon the spectrum plate may be long and apparently symmetrical. Since the inherent difficulties of the measurement of such lines are great, and the sources of error so many and insidious, we have not included these lines among those which we recommend for standards. The rejection of these lines from the series of standards would enlarge the region near λ 5800 for which the Committee has recommended that barium lines be used. In a still higher degree, the same reasoning holds in the case of the lines of group *d* and the still greater difficulties of measurement render them in our judgment unsuitable for standards of the third order.

LINES GREATLY DISPLACED TO THE VIOLET UNDER PRESSURE

Group e.—When the data from the Mount Wilson and Pasadena plates were finally assembled, some lines were noticed for which the mountain plates gave larger values than the valley plates. The quality of these lines is poor, and it was at first thought that the errors of measurement were proving greater than was expected; but it soon appeared probable that the differences between the mountain and the valley wave-lengths were real, and that the lines constitute a group the members of which are widened and greatly displaced to the violet under pressure, and therefore to the red under the lessened pressure at the elevation of Mount Wilson. In continuation of the grouping scheme of Gale and Adams, we have formed the group *e*, whose definition has already been given. On inspection of the plates of Gale and Adams taken under a pressure of 9 atmospheres with a comparison spectrum at atmospheric pressure, which they kindly placed at our disposal, these lines are seen to be enormously widened with the maximum displaced to the violet. They appear as though blown to the violet under this pressure without a maximum sufficiently pronounced for measurement, but the direction and character of the displacement can be determined with certainty by inspection.

These lines occur in the yellow-green and the red. The mean of the differences, Pasadena − Mount Wilson, is −0.014 Ångström. On the assumption that the lessened wave-length in Pasadena is due to the difference in atmospheric pressure, the displacement relative to the standards is −0.070 Ångström per atmosphere. The mean displacement of the standards per atmosphere is +0.006 Ångström; hence the absolute displacement is −0.070 Ångström +0.006 Ångström = −0.064 Ångström. This would mean an enormous displacement for these lines, and there has been as much hesitancy in accepting displacements to the violet under pressure even to a small amount, as there has been in accepting displacements to the red of the order of those which the measurements given in this paper appear to show.

Since the lines showing these great displacements are unsymmetrically widened under pressure, in the case of group *d* to the red and of group *e* to the violet, it has been thought by others, notably Fabry and Buisson,[1] that such displacements were only apparent and arose from the difficulties inherent in the comparative measurement of such lines. Widening and displacement under pressure are intimately connected, and it is exceedingly difficult to determine whether there is actual displacement of a line, when the widths of the line are different under the two conditions of pressure. To render our data so far as possible free from such errors, and to guard against their entering into the differences, Pasadena − Mount Wilson, in such a way as to produce or increase these differences we measured the widths of the lines and based the comparison only upon plates for which the widths of the lines on the mountain plates equaled or slightly exceeded the widths of the corresponding lines on the Pasadena plates. The measurements of the widths are given in the seventh and eighth columns of Table VI.

As to the lines showing the enormous displacements to the red, the results of this paper are in accord with the measurements of Gale and Adams, and no doubt of their reality could remain in the mind of anyone who has had opportunity to examine the Gale and Adams plates, even if careful measurements had not been made. The mean width on the mountain plates of the lines showing dis-

[1] *Comptes rendus*, **148**, 688, 1240, 1909.

placement to the violet under pressure is 0.131 Ångström and on the Pasadena plates, 0.119 Ångström. The increase of width in the case of such lines is unsymmetrical, whether the increase is due to increased pressure or to greater density of the radiating vapor, and is largely on the violet side of the line. The mean width of the lines of this group on the mountain plates exceeds the mean width on the Pasadena plates by 0.012 Ångström, so that in the absence of pressure-shift, the wave-lengths should have been less on the mountain than in the valley by approximately 0.006 Ångström. As a matter of fact we found them longer by 0.014 Ångström. On the assumption of no pressure displacement to the violet this would imply an average error of 0.020 Ångström in the measurement of lines whose mean width is 0.120 Ångström. The mean deviations averaged 0.006 Ångström for the Pasadena plates and 0.004 Ångström for the mountain plates, with mean probable errors of 0.0017 Ångström and 0.0011 Ångström respectively. The mountain and valley plates were measured by the same observers, reduced by the same methods, the lines referred to the same standards, and the plates intermingled in the course of the measurement, so that the effects due to systematic error and personal equations were eliminated as completely as possible. In view of the above considerations, and the degree of precision obtained in the case of normal lines, there seems sufficient ground for adopting the direct interpretation of the data, viz., that the lines of groups *d* and *e* are greatly displaced as well as unsymmetrically widened by pressure, and to the red and violet respectively.

RELATION BETWEEN WAVE-LENGTHS IN ARC AND IN ROWLAND'S TABLE

Reference has been made at various times in this paper to the differences, Rowland—Pasadena. The subject deserves a somewhat fuller discussion. The relation has been of service in the present investigation in classifying the lines into groups which agree with those formed upon the basis of pressure-shift. In the comparison we have used the wave-lengths in Pasadena in the case of the lines in groups *d*, *e*, and Mn; for the lines of groups *a*, *b*, and sub *d*, we have used the means of the mountain and valley plates. The differences, Rowland—Kayser, show the same classification, as appears from a consideration of Table X.

That there is a physical basis for the classification cannot be doubted. This is particularly evident in the case of groups *d*, *e*, and Mn. It is not so evident from the Table in the case of groups *a*, *b*, and sub *d*, but it becomes evident when their respective pressure-shifts 0.0036 Ångström, 0.009 Ångström, 0.019 Ångström, are taken into account. It would be interesting to discuss these results somewhat fully from the point of view of some solar problems, but such a discussion would lead far afield, and investigations now in progress are furnishing so much additional material bearing upon the subject that any discussion would be premature.

TABLE X

ROWLAND DIFFERENCES

	a	*b*	*d*	Sub *d*	*e*	Mn
S. and W.	0.214	0.210	0.191	0.215	0.239	0.189
Kayser	0.215	0.207	0.195	0.213	0.233	0.196

Much has been said about the irregular and apparently capricious differences that are obtained in comparing the wave-lengths of Rowland's table with wave-lengths in the arc, and efforts have been made to determine some factor or operator by which the Rowland wave-lengths can be converted into international units. It is clear from Table X that the apparently large irregularities in the Rowland differences are not inaccuracies, but represent real differences. This is clearer still when one considers that the lines in groups *d*, *e*, and Mn are nearly all sharp lines in the solar spectrum, and hence the accidental errors in Rowland's table in the case of these lines are a minimum. It is equally clear that no one factor can be found by which the wave-lengths can be converted from one system into the other with satisfactory accuracy, even for a limited region of the spectrum. This late justification of Rowland's values for individual lines has of course no bearing upon the systematic errors pointed out by Fabry and Perot.[1] In fact the curve plotted from the data in Table VII shows the same course as theirs. The method proposed by them for passing from one system to the other is alone capable of yielding satisfactory results.

[1] *Astrophysical Journal*, **15**, 272, 1902.

LINES RECOMMENDED FOR STANDARDS

The criterion applied in selecting the lines suitable for use as standards of the third order has been the close agreement of the wave-lengths as determined from the mountain and from the Pasadena plates. The lines yielding the same values from the two series of plates either belong to the same group as the standards, or undergo practically the same displacement for small changes in pressure. The lines examined are separated into three sections, namely: λ 5371–λ 5563 referred to the flame-line standards, λ 5569–λ 5658 referred to the standards of sub-group *d*, and λ 5975–λ 6494 referred to standards belonging to group *b*. The second section has already been considered and for the reasons given omitted from the list of recommended lines. Two groups of lines, *d* and *e*, occurring in the first and third sections, have also been dropped from the list of available lines because of their extreme sensitiveness to changes in the pressure and density of the emitting vapor, manifesting itself in displacement and great dissymmetry. There remain only 25 lines which we are able to recommend as tertiary standards. The distribution of these lines in reference to the 17 international standards of the second order may be seen in Table VII. The results of our examination of 100 lines are as follows:

Recommended	Standards of the second order	17
	Standards of the third order	25
Rejected	Lines of group *d*	17
	Lines of sub-group *d*	9
	Lines of group *e*	15
	Lines of manganese	3
Questionable		14
		100

We have not been able to measure the many lines of intensity 2 of Kayser's list, as they either did not appear on our plates, or were too faint for the purpose of measurement. Of the 14 lines in the questionable list, a large proportion will probably prove available and be extremely useful as faint-line standards when their wave-lengths are definitively determined; but our material was so insuffi-

cient and the quality of the lines so poor that we do not include them in the list of recommended standards on the basis of our measurements even though the precision is apparently high (mean p.e. 0.0016 Ångström for the Pasadena plates, p.e. 0.0011 Ångström for the mountain plates). It would appear from the agreement of the mountain and valley measurements that the lines belong to the same groups as the standards to which they were referred but in four instances this is not the case. The pair of lines λ 5462–5463 were found by Adams[1] to be much stronger in the flame than in the core of the arc, and therefore would appear to belong to the group of flame lines, but under a pressure of 9 atmospheres their appearance differs widely from the appearance of the flame lines proper. They seem related to the pair λ 6102–6103. In the normal arc they are all hazy, and under a pressure of 9 atmospheres they appear enormously diffused with no indication of displacement.

THE PRECISION OF THE MEASUREMENTS

The precision that may be obtained by the use of a plane grating is best shown by the agreement between the wave-lengths obtained from the two series of plates in the case of lines belonging to the same group as the standards used in their measurement. In Table VII the measurements of such lines are given. The mean of the differences, Pasadena − Mount Wilson, taken without regard to sign, is 0.0010 Ångström. This seems to us a more satisfactory measure of the precision obtained than the mean probable errors of 0.0007 Ångström and 0.0006 Ångström in the case of the valley and mountain plates respectively. As these measurements are absolute and not differential, the precision may be taken as representing the degree of accuracy given by the plane grating in interpolating between the international standards of the second order in the case of lines of good quality. This is the same order of precision obtained by one of us in a previous investigation[2] when the conclusion was drawn that with homogeneous standards

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 126, 1909.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

an accurracy of 0.001 Ångström could be obtained in the case of the best lines.

SOME RELATED CONSIDERATIONS

In comparing our results with those published by Kayser, systematic differences appear, for some of which we can find no explanation. In the case of the lines of groups *b*, sub *d*, and the three lines of manganese, our values from the Pasadena plates are larger than his in the mean by 0.0037 Ångström; in the case of the lines of group *e*, our values from the Pasadena plates are smaller in the mean by 0.0056 Ångström. As these lines show more or less dissymmetry, the first group widening more on the red edge, the latter on the violet edge when the pressure or density of the emitting vapor is increased, such systematic differences may represent the personal equations of the observers in setting upon lines of this character, or may depend upon the different appearances that the two types of gratings would give to such lines.

There appear systematic differences between our results and those of Kayser in the case of lines belonging to groups *a* and *b*, for which our two series of plates show a high degree of agreement. These differences persist after allowance has been made for the different values of the secondary standards to which the tertiaries are referred in the two instances. For the lines referred to the flame-line standards of group *a*, λ 5371–λ 5463, Kayser−S. and W.= +0.0003 Ångström; for the lines of group *b*, λ 5975–λ 6355, Kayser−S. and W.=+0.005 Ångström; for the two lines, λ 6421 and λ 6462, the differences are accounted for by the large difference in the wave-length of the standard λ 6430. Thus at the beginning and end of the list the values agree, but in the intermediate region the difference is larger than it seems possible to assign to accidental errors. In a paper published during the preparation of this article for the press, Goos[1] reports similar systematic differences between his values and those of Kayser for the blue-green region which he has investigated by means of a plane grating, and thinks that these deviations are to be explained by the fact that Kayser used a simple linear interpolation between each two of the secondary standards and then averaged the results.

[1] *Astrophysical Journal*, **35**, 229–230, 1912.

The result of his previous investigation of the conditions which influence the dispersion in the case of the concave grating, and which should be taken into account is stated as follows:

> Es ist bedenklich, bei Konkavgitterspektrogrammen, selbst innerhalb enger Intervalle, linear ohne Anbringung von Korrektionen zu interpolieren, da man nicht im voraus weiss, in welchem Bereich der Platte die Konstanz des Maasstabes ausreichend ist.[1]

In practice the interpolations in the case of the concave grating appear fully as troublesome as in the case of the plane grating, when a high degree of precision is required. With the plane-grating spectrograph of long focus, the dispersion is apparently more nearly linear than it is normal with the concave grating, and between linear and normal dispersion there is not much to choose on the ground of convenience in interpolation. There seems to be no good reason for assuming a priori that the precision is in one case higher or lower than in the other. It appears that in either case the dispersion for each plate must be considered by itself, and its departure from the normal or linear character must be determined in order to fix the interval through which the dispersion is normal or linear to the required degree.

The idea that changes in atmospheric pressure, such as occur in passing from sea-level to the altitudes of modern observatories, may alter the relative wave-lengths of spectrum lines by measurable amounts introduces a new point of view, and suggests other conditions that a series of standards should fulfil when the highest precision is to be obtained. The red region of the spectrum which is especially rich in lines showing great displacements had not been systematically studied before Gale and Adams undertook its examination. In view now of the large amount of observational data accumulated, recognition should be given to the possibility and the probability of introducing errors by neglecting the effects of relative pressure displacement. Even in the case of some standards of the second order the change in relative wave-lengths amounts to 0.003 Ångström on Mount Wilson, and for observatories of greater elevation it may reach 0.005 Ångström. For other

[1] *Zeitschrift für wissenschaftliche Photographie*, **10**, 206, 1911.

iron lines the change is 0.010 Ångström on Mount Wilson, and may reach 0.015 Ångström at the higher stations. There are a number of important observatories at which such changes in wave-length would prove disturbing.

OBSERVATORIES AT HIGH ALTITUDES

Heidelberg, Germany	570 m	Astronomical and Astrophysical Institute, Königstuhl
Berne, Switzerland	573	Observatory of the Cantonal University
Madrid, Spain	655	Astronomical and Meteorological Observatory
San Luis, Argentina	800	Southern Observatory of the Carnegie Institution
Mount Hamilton, California	1283	Lick Observatory
Denver, Colorado	1650	Chamberlain Observatory
Mount Wilson, California	1794	Solar Observatory of the Carnegie Institution
Flagstaff, Arizona	2210	Lowell Observatory
Tacubaya, Mexico	2280	National Observatory of Mexico
Palani Hills, India	2347	Kodaikánal Observatory
Arequipa, Peru	2452	Station of Harvard College Observatory
Bogota, Republic of Colombia	2634	National Observatory of Republic of Colombia
Quito, Ecuador	2908	National Observatory of Ecuador

GENERAL SUMMARY

1. The precision obtainable with the plane-grating spectrograph in determining the absolute wave-lengths of standards of the third order by interpolation between the international standards is 0.001 Ångström in the case of good lines. Two entirely different instruments were used and the mean of the differences between the wave-lengths in the two series is 0.0010 Ångström, a quantity less than the sum of the probable errors. This represents the performance of two similar instruments; whether the actual wave-lengths were determined with this degree of precision or whether there is some obscure systematic error in the methods will appear when more determinations are made by other observers with other instruments.

2. The secondary standards between λ 5371 and λ 6494, except λ 5763, were examined in reference to their relative wave-lengths. The conclusion is that there are no errors in their relative wave-lengths exceeding 0.001 Ångström, except in the case of the line λ 5434, where it reaches 0.002 Ångström.

3. Of the 100 lines studied in the region λ 5371–λ 6494, we find 17 thoroughly suitable for secondary standards, and 25 for tertiary standards; 44 are discarded because of abnormal pressure displacements accompanied with dissymmetry; and 14 are provisionally classified as of questionable utility.

4. In comparing the wave-lengths from the two series of plates, Mount Wilson and Pasadena, it was found that in the case of 20 lines the wave-lengths determined from the valley plates were in the mean 0.009 Ångström longer than the wave-lengths determined from the mountain plates when lines of the same width were compared. All of these lines, that are of sufficient intensity to appear upon the Gale and Adams plates at a pressure of 9 atmospheres, show strongly unsymmetrical widening and large displacements of the maxima to the red. They seem to us inherently unsuitable for standards and therefore have been rejected.

5. It likewise appeared from the comparison of Mount Wilson and Pasadena determinations of wave-lengths that 15 lines gave values on the valley plates shorter than on the mountain plates by 0.014 Ångström, when the comparison was made between lines of equal widths. Upon the plates of Gale and Adams taken at 9 atmospheres, these lines are very unsymmetrically widened and the diffuse maxima displaced to the violet. There seems sufficient reason for considering them a distinct group, the components of which are really displaced to the violet and unsymmetrically widened under pressure. The reasons for rejecting the lines showing great displacement to the red hold in even greater degree for these lines.

6. From the large differences that appear in the amounts by which different lines are displaced under pressure, it would be desirable to know the pressure displacements of all lines that are finally accepted as standards of the second or third order. The secondary standards λ 5569–λ 5658 are a case in point, the differences between the displacements of these lines and the neighboring standards being 0.015 Ångström per atmosphere. The difficulties of exact measurement due to their lack of sharpness and to their dissymmetry are so great that we are forced to omit them from the list of recommended lines. In the selection of barium lines to

supplement the iron lines, a preliminary examination of the barium lines with regard to displacements under pressure is to be recommended; and in extending the series of standards to the red, caution should be especially exercised in the selection of lines, in view of the occurrence in the red of lines showing great pressure displacements and the rapid increase of displacement with wave-length.

MOUNT WILSON SOLAR OBSERVATORY
May 1912

ON THE DETERMINATION OF THE ORBITAL ELEMENTS OF ECLIPSING VARIABLE STARS. II[1]

By HENRY NORRIS RUSSELL

PART III. ECCENTRICITY OF ORBIT

§ 5. *Determination of the elements.*—The problem is now completely solved for a circular orbit. But analogy with the orbits of spectroscopic binaries of similar period leads us to expect that the actual orbits of eclipsing variables will often show a small eccentricity (rarely exceeding 0.1); and this is confirmed in several cases by displacement of the secondary minima from the position midway between the primary minima, or by spectroscopic observations of the variables.

In such a case the elements computed from the light-curve as above on the assumption of a circular orbit will differ from the true values. If, however, the eccentricity is small enough to justify the neglect of its square, simple equations can be found which express very approximately the relation between the true and computed elements.

Let us consider the brighter body as moving about the fainter (for convenience in comparison with spectrographic results). Let e be the eccentricity of the orbit, and ω the longitude of periastron (counted as usual from the ascending node in the direction of motion), r the radius vector, v the true, and M the mean anomaly. Then the projected distance of centers of the stars is given by the equation

$$\delta^2 = r^2\{\cos^2 i + \sin^2 i \cos^2 (v+\omega)\} . \qquad (23)$$

We have also, neglecting e^2 and taking the major axis of the orbit as unity,

$$r = 1 - e \cos M , \qquad v = M + 2e \sin M .$$

If now we set $\theta = M + \omega - \frac{\pi}{2}$, θ will be an angle increasing uniformly with the time, which when $e = 0$ becomes identical with the angle

[1] Continued from *Astrophysical Journal*, **35**, 340, 1912.

called θ in Part I. It may be defined as the mean longitude, measured from a point 90° in advance of the node. Introducing this into the above expressions, and for brevity setting $e \sin \omega = g$, $e \cos \omega = h$, we find

$$r = 1 - g \cos \theta + h \sin \theta , \qquad v + \omega = \frac{\pi}{2} + \theta + 2h \cos \theta + 2g \sin \theta .$$

Introducing these into (23) and neglecting terms of the second and higher orders in the small quantities g and h, we find after some reductions

$$\delta^2 = (1 - 2g \cos \theta) \cos^2 i + (1 + 2g \cos \theta) \sin^2 i \sin^2 \theta + 2h \sin \theta (1 + \sin^2 i \cos^2 \theta) .$$

If now we set

$$\theta' = \theta + h \, (\operatorname{cosec}^2 i + 1) , \tag{24}$$

and again neglect terms of the second order, this becomes

$$\delta^2 = (1 - 2g \cos \theta') \cos^2 i + (1 + 2g \cos \theta') \sin^2 i \sin^2 \theta' + 2h \sin \theta' (1 - \cos \theta')(1 - \sin^2 i \cos \theta') .$$

Setting

$$A \sin \gamma = (1 + g \cos \theta') \sin i ,$$
$$A \cos \gamma = (1 - g \cos \theta') \cos i ,$$

whence

$$\left.\begin{aligned} A &= 1 + g(1 - 2 \cos^2 i) \cos \theta' , \\ \cot \gamma &= (1 - 2g \cos \theta') \cot i , \end{aligned}\right\} \tag{25}$$

we find

$$\delta^2 = A^2 (\cos^2 \gamma + \sin^2 \gamma \sin^2 \theta') + 2h \sin \theta' (1 - \cos \theta')(1 - \sin^2 i \cos \theta') . \tag{26}$$

This expression is a minimum when $\theta' = 0$; that is, $\theta' = 0$ at the middle of eclipse. If the epoch of this phase is t_1, we have at any time t

$$\theta' = \frac{2\pi}{P} (t - t_1) , \tag{27}$$

which is identical with equation (8) defining the angle θ employed in the computation of elements for a circular orbit. For the secondary minimum we may set

$$\theta'' = \theta + \pi - h(\operatorname{cosec}^2 i + 1) , \tag{28}$$

and find in the same fashion

$$\left.\begin{aligned}\delta^2 = B^2\,(\cos^2\beta + \sin^2\beta\,\sin^2\theta'') \\ -2h\,\sin\theta''(1-\cos\theta'')\;(1-\sin^2 i\,\cos\theta'')\,,\end{aligned}\right\}$$

where

$$\left.\begin{aligned}B = 1-g(1-2\,\cos^2 i)\,\cos\theta'', \\ \cot\beta = (1+2g\,\cos\theta'')\,\cot i\,, \\ \theta'' = \frac{2\pi}{P}(t-t_2)\,,\end{aligned}\right\} \quad (29)$$

where t_2 is the epoch of middle of secondary minimum. In these equations nothing has been neglected except the square of the orbital eccentricity and terms depending upon it.

The last term in equations (26) and (29) is very small unless θ attains large values. Neglecting it for the moment, we see that the light-curve during either eclipse may with very considerable approximation be regarded as symmetrical, and identical with that resulting from the mutual eclipse of two stars of the same radii r_1 and r_2 as the actual stars, moving in a circular orbit which for the principal eclipse must be assumed to be of radius A and inclination γ, and for the secondary eclipse of radius B and inclination β.

These quantities contain variable terms of the form $g\,\cos\theta'$. Since θ' vanishes at the middle of the eclipse, the changes in these terms will be small, and we may with considerable approximation replace $\cos\theta'$ by its mean value η during the eclipse, and A and γ by their corresponding mean values A_m and γ_m. If ζ is the value of θ' at beginning or end of eclipse, we have with sufficient approximation $3\eta = 2+\cos\zeta$.

The observed light-curve during principal minimum will therefore differ very little from one computed for a circular orbit with inclination γ_m, radius A_m, and stars of radii r_1 and r_2, or, since the linear dimensions of the system may be changed in any ratio without affecting the light-curve, by an orbit of radius unity, inclination γ_m, and stars of radii $\frac{r_1}{A_m}$ and $\frac{r_2}{A_m}$. It follows at once that, if we determine the circular elements which best represent the observed light-curve by the methods already developed (assuming the radius of the orbit to be unity), the ratio $k = \frac{r_2}{r_1}$ will be given by

this calculation with approximate accuracy, and the other elements will closely approach the values:

$$\left.\begin{array}{ll} \text{Principal Minimum:} & \text{Secondary Minimum:} \\ r'_1 = r_1(1-g\eta)\,, & r''_1 = r_1(1+g\eta)\,, \\ \cot i' = \cot i\,(1-2\,g\eta), & \cot i'' = \cot i(1+2g\eta), \end{array}\right\} \quad (30)$$

where

$$g = e \sin \omega\,, \qquad 3\eta = 2+\cos \zeta\,.$$

These equations involve only $e \sin \omega$. To this degree of approximation the other component of the eccentricity, $h = e \cos \omega$, affects only the interval between the primary and secondary minima. From the equations (24)–(29) we find at once

$$h\ (\operatorname{cosec}^2 i+1) = \frac{\pi}{P}(t_2-t_1-\tfrac{1}{2}P)\,, \qquad (31)$$

a very well known relation, which suffices to determine $e \cos \omega$ with high accuracy.[1]

The determination of $e \sin \omega$ is less easy and in practice somewhat less exact. The equations (30) cannot often be used as they stand, for the secondary minimum is not usually deep enough to permit an independent determination of the elements from its light-curve. We must distinguish two cases, according as the principal minimum does or does not show a constant phase.

In the first case k, r'_1, and i' are determined for the primary minimum in the usual way, assuming a circular orbit. We have then from (30)

$$r''_1 = r'_1(1+2g\eta)\,, \qquad \cot i'' = \cot i'(1+4g\eta)\,. \qquad (32)$$

Assuming different values of g, the light-curve of the secondary minimum may be computed and the value of g found which best represents the observations. Only the times of the four contacts need be computed in the first approximation, and the work involved is small. If the principal eclipse occurs near periastron, and is nearly grazing, the secondary eclipse may be partial.

If the principal minimum shows no constant phase the problem is more complicated, for the method previously used for finding the elements is inapplicable, as it involved the assumption that

[1] C. André, *Traité d'astronomie stellaire* (Paris, 1900), p. 241.

the eclipsed area is the same at the two minima. In the present case we have to regard the percentages of area eclipsed at each minimum as unknown, a_1 and a_2. Between these quantities and the light-intensities λ_1 and λ_2 at the two minima, there exists the relation

$$\frac{1-\lambda_1}{a_1}+\frac{1-\lambda^2}{k^2a^2}=1\,. \tag{33}$$

The subscript 1 here refers to the eclipse of the smaller star by the larger. Whether this is the principal or the secondary minimum must be determined by trial.

From the light-curve for the principal minimum we can derive a relation between k and a_1 (or a_2, as the case may be) of the form

$$\chi(k, a_1, \tfrac{1}{4})=\text{const.} \tag{34}$$

Assuming any value of k we may find a_1 from Table III and then a_2 from (33). If the light-curve of the secondary minimum is sufficiently well defined to determine $\chi(k, a_2, \frac{1}{4})$ with precision, the comparison of the computed and observed values of this function will suffice to determine the correct value of k.

Otherwise we may compute the actual light-curve for the secondary minimum for assumed values of k, proceeding as follows. Having found a_1 and a_2 as above, we take out from Table I the values p_1 and p_2, corresponding to k, a_1 and k, a_2. The projected distances between the centers of the stars at the middle phases of the two minima are then

$$\delta_1=r_1(1+p_1k)=(1-g)\cos i\,,$$
$$\delta_2=r_1(1+p_2k)=(1+g)\cos i\,,$$

whence

$$g=\frac{(p_2-p_1)k}{2+(p_1+p_2)k}\,. \tag{35}$$

We may now derive the circular elements r'_1 and i' which best represent the principal minimum by the equations (20) (Part I), and then, by equations (32), find the elements r''_1 and i'' which will best represent the secondary minimum on the assumption of a circular orbit. From these the light-curve (or in the first trials merely the times of beginning and end of eclipse) may be computed.

The whole sequence of these operations can be completed, with the aid of the tables, in a few minutes, so that the process of finding the value of k which gives the best light-curve for the secondary minimum is not so long as it may appear. It is of course well to start with the elements obtained on the assumption of a circular orbit. A few trials will suffice to determine within what limits an acceptable value of k must lie. It may happen that these limits are wide; in which case the problem is nearly or quite indeterminate.

§ 6. *Asymmetry of the light-curve.*—We have still to consider the final term of equation (26), which, changing sign with θ', evidently gives rise to a lack of symmetry between the ascending and descending branches of the light-curve. Suppose that the same brightness (that is, the same value of δ) is reached for $\theta'=\theta_0+\epsilon$ and $\theta'=-\theta_0+\epsilon$. We then find, easily, neglecting terms of the second order,

$$\epsilon=-h(\sec\theta_0-1)(\operatorname{cosec}^2 i-\cos\theta_0)\,, \tag{36}$$

which gives the displacement of the mean epoch of two corresponding phases of ascending and descending light whose interval is $2\theta_0$. The displacement of the secondary minimum, relative to the epoch following the primary by half a period, is in the same units, by (31),

$$\Delta=\frac{2\pi}{P}(t_2-t_1-\tfrac{1}{2}P)=2h(\operatorname{cosec}^2 i+1)\,,$$

whence we have

$$\frac{\epsilon}{\Delta}=-\frac{(\sec\theta_0-1)(\operatorname{cosec}^2 i-\cos\theta_0)}{2(\operatorname{cosec}^2 i+1)}\,. \tag{37}$$

This is always a small fraction. If we set $i=70°$, $\theta_0=45°$ (values well outside those found ordinarily in practice), this ratio becomes 0.04. If $\theta_0=30°$, it is 0.01. The asymmetry of the light-curve is therefore always very small compared with the displacement of the secondary minimum, and in fact is usually too small to be detected by observation—another well-known result (André, *Traité d'astronomie stellaire*, **2**, 248). The negative sign in (37) denotes that the duration of the light change is longer on that side of the minimum which lies nearest to the adjacent secondary minimum.

On account of this, and also because of other neglected terms, the light-curve computed rigorously from elliptical elements obtained as above will not agree exactly with that derived from the preliminary circular elements, and may not represent the observations as well as the latter, though the deviations will usually be small. In such a case we may obtain a satisfactory approximation by starting with a fictitious light-curve which deviates at every point as far, and in the same sense, from that computed from the circular elements as the empirical light-curve which best represents the observations deviates from that computed from the elliptic elements. Representing this as well as may be by new circular elements, we shall be led to new elliptic elements, which will obviously give a light-curve agreeing very closely with the observed curve. Such refinements of calculation will obviously be very rarely necessary; but their introduction will take much less time than an attempt to correct the elements by differential methods, which will seldom if ever be advisable.

The problem of determining the elements, when the stars are supposed to present uniformly illuminated circular disks, is now completely solved.

PART IV. ELLIPTICITY OF THE STARS

§ 7. *General theory.*—In certain eclipsing variables—the classic example being *β Lyrae*—the light-changes are continuous, and the brightness is sensibly greater half-way between minima than just before or after eclipse.

This phenomenon is reasonably explained by the assumption that the stars are elongated by their mutual attraction, a hypothesis which is the more probable, since in such systems the component stars usually appear to be separated by a distance not greatly exceeding their own diameters.

The researches of Darwin[1] indicate that the figure of such stars is probably nearly ellipsoidal, with the longest axis in the line joining the components, and the shortest perpendicular to the orbit plane.

[1] *Philosophical Transactions*, A **206**, 160–248, 1906.

If the components are equal in mass and density, they will be similar in form. Otherwise the smaller mass will be the more elongated unless its density considerably exceeds that of the other. In the absence of data bearing on this question, and to avoid what are practically unsurmountable difficulties of calculation, we will assume (along with those who have previously discussed the problem)[1] that the components are similar and similarly situated ellipsoids whose longest axes coincide with the line joining their centers; and also, as in all our work so far, that they appear to the observer as uniformly illuminated disks.

A. Roberts has pointed out[2] that in such a case we cannot hope to find the degree of flattening of the ellipsoids in the direction of their polar axes. A general proof may be given as follows.

The lines of sight which touch the surfaces of the two ellipsoids form two elliptical cylinders with parallel axes, which may wholly or partly overlap one another. The apparent disks are obtained by cutting these cylinders by a fixed plane perpendicular to the line of sight. If we cut them by any other fixed plane, we obtain ellipses whose areas (including the area of any eclipsed segment) bear a fixed and constant ratio to the areas of the apparent disks, so that these latter areas may be used instead of the apparent disks in computing the amount of light which the distant observer receives.

Let us now choose the plane of the relative orbit as our fixed plane and consider two systems, one of which may be obtained from the other by increasing all the co-ordinates perpendicular to the orbit-plane (including that of the distant observer) in any constant ratio, while leaving all co-ordinates unaltered which are measured parallel to the orbit-plane. The areas in the orbit-plane, which may be used as a measure of the light received from the system, are identical in these two cases. Hence, by a proper choice of the assumed surface-brightness of the stars, we may obtain an absolutely identical light variation in the two cases. The two systems of elements will differ only in that the polar axes of the two ellipsoids, and the tangent of the angle between

[1] G. W. Myers, *Astrophysical Journal*, **7**, 8, 1898.

[2] A. Roberts, *Monthly Notices*, **63**, 528, 1903. .

the orbit-plane and the line of sight (or cot i as ordinarily defined) will all be changed in an arbitrary constant ratio.

We may in general choose this constant so as to make one of our ellipsoids a prolate surface of revolution, with its longer axis coincident with the line of centers, and, since we have already assumed the two ellipsoids to be similar, this will be true of the other also.

Such a system of two prolate spheroids will be the basis of our calculations. It is physically an impossible one, for all actual figures of equilibrium which are elongated toward one another by mutual attraction must also be flattened at the poles owing to their axial rotation (whose duration must coincide with the orbital period). When we have found the extent of the elongation in the equatorial plane, the amount of the polar flattening may be at least approximately estimated. If a, b, c are the axes of the ellipsoid, in descending order of magnitude, we should have, for nearly spherical bodies

$$b-c=\tfrac{2}{3}(a-b)\,.$$

For more elongated bodies, Darwin's results for equal masses of homogeneous, incompressible fluid, may be used as a guide. They may be represented, with remarkably close approximation, by the empirical formulae

$$\frac{b^2}{a^2}=\frac{1-4.4a^3}{1+3.1a^3}\,,\qquad \frac{c^2}{a^2}=\frac{1-3.4a^3}{1+8.6a^3} \tag{38}$$

(when the distance of centers is taken as the unit of length), as is illustrated by the following examples, which are either taken from Darwin's work, or computed by means of his equations.

a	$1-\frac{b^2}{a^2}$		$1-\frac{c^2}{a^2}$	
	From Darwin's Formulae	From (38)	From Darwin's Formulae	From (38)
0.099	0.008	0.007	0.012	0.012
.204	.059	.062	.098	.095
.284	.153	.161	.223	.232
.340	.260	.263	.352	.353
.410	.425	.425	.517	.520
.497	.664	.668	.719	.720

The value $a=0.340$ marks, according to Darwin, the limit beyond which these figures of equilibrium, if composed of homogeneous, incompressible fluid, become unstable. This assumption differs so widely from the conditions prevailing in the stars that the actual degree of ellipticity, and the incidence of instability, may differ considerably from Darwin's figures; but the latter are nevertheless the best guide in existence in this very intricate matter.

In any event, if we have obtained a satisfactory representation of our light-curve on the assumption of stars of prolate spheroidal form, we have only to diminish the computed polar axis in the ratio which seems most plausible, according to the above, and at the same time diminish cot i in the same ratio, to obtain a system of elements, giving the same computed light-curve, and satisfying also the physical condition under consideration.

Taking up the problem in detail, let a_1, b_1 be the major and minor axes of the larger spheroid, and ϵ the eccentricity of the meridian section, and let $a_2=ka_1$, $b_2=kb_1$ be the axes of the smaller ellipsoid. The two stars will appear as elliptical disks, whose major axes coincide with the line of centers. Their minor axes will always be b_1 and b_2, but their major axes will usually be less then a_1 or a_2. If their values at any instant are d_1 and d_2, and ϕ is the angle between the line of centers and the line of sight, we find readily $d_1^2=a_1^2(1-\epsilon^2\cos^2\phi)$, $d_2=kd_1$. But, with our previous notation, $\cos\phi=\sin i\cos\theta$, whence

$$d_1^2=a_1^2(1-\epsilon^2\sin^2 i\cos^2\theta)\,. \tag{39}$$

Suppose now that the apparent distance between the centers of the disks is δ. If $\delta>(1+k)d_1$ there will be no eclipse, and if $\delta<(1-k)d_1$ the eclipse will be total. If δ lies between these two limits, the eclipse will be partial—the two elliptical disks overlapping in the direction of their major axes.

We will not modify the ratio of the eclipsed portion to the areas of the two disks, if we alter the dimensions of the ellipses at right angles to the line of centers, i.e., their minor axes, in any constant ratio. By choosing this ratio equal to $\frac{d_1}{b_1}$ we can make the disks circular[1] and of radii d_1 and kd_1. We can now use the

[1] G. W. Myers, *op. cit.*, p. 10.

notation of our previous work, the only change being that d_1 takes the place of r_1. We then have

$$\delta = d_1\phi(k, \alpha) = d_1\{1 + kp(k, \alpha)\}, \tag{40}$$

where α is the percentage of area of the smaller disk which is eclipsed. Let l_1 and l_2 be the amounts of light which would reach us from each star if there were no eclipse. The light actually received will be $l_1 + (1-\alpha)l_2$. The quantities l_1 and l_2 are not constant, but vary with the varying area of the apparent disks. If L_1 and L_2 represent their maximum values, which are obviously reached when $\theta = 90°$, we have

$$\frac{l_1}{L_1} = \frac{l_2}{L_2} = \frac{d_1}{a_1} = (1 - \epsilon^2 \sin^2 i \cos^2\theta)^{\frac{1}{2}},$$

whence, if l is the actual amount of light received by us at any time,

$$l = l_1 + (1-\alpha)l_2 = (1 - \epsilon^2 \sin^2 i \cos^2\theta)^{\frac{1}{2}}[L_1 + (1-\alpha)L_2]. \tag{41}$$

When $\epsilon = 0$ this reduces to the familiar formula for spherical stars. For brevity, let us now set

$$\epsilon^2 \sin^2 i = z. \tag{42}$$

§ 8. *Determination of the ellipticity and other elements.*—We have now before us two problems, the determination of this constant z, defining the effective ellipticity of the stars, and that of the other elements when this is known. Various methods have been proposed for the solution of the first problem in special cases. Roberts[1] has published tables which give both ϵ and i, on the assumption of equal stars revolving in contact, and Von Hepperger[2] has developed a very ingenious method, which, however, is available only when the two minima are of unequal depth.

It does not appear to have been previously observed that a satisfactory determination of z can be very simply made by plotting the observed values of l^2 against $\cos^2\theta$. Since the second factor of (41) is constant when there is no eclipse, the resulting points for values of θ near 90° will be on a straight line whose

[1] *Monthly Notices*, **63**, 535, 1903.

[2] *Sitzungsberichte der Akademie der Wissenschaften in Wien*, Math.-Naturwiss. Kl., **118**, 933, 1909.

slope gives the desired value of z. When eclipse begins the observed points fall below this straight line and lie on a descending curve.

This method might seem to fail when the stars are in actual contact, and more or less of their area is eclipsed, except when $\theta = 90°$, so that the curve above described has no rectilinear portion; but in this case the eclipsed area is very small when θ is near 90° (varying, as may be easily shown, approximately as $\cos^3\theta$). The tangent to the observed curve at the point for which $\cos^2\theta = 0$ can be drawn, and this gives as before the value of z.

Having found z, the light-curve may be "rectified," removing all apparent influence of the ellipticity of the stars. From equation (41), and the logarithmic nature of stellar magnitude, it follows that the actual change in the visual magnitude of the system at any time, as compared with the maximum, may be found by adding the changes in magnitude due to the two factors, one of which arises wholly from eclipse, and the other from ellipticity. By subtracting from the observed magnitudes the computed variation due to the latter cause, we may obtain a light-curve of the ordinary "*Algol*" form, with constant light between eclipses, which represents the variations in brightness due to eclipse alone.

From this curve it may be determined whether the eclipses are total or partial, and the magnitudes and the values of θ corresponding to assigned values of a_1 (for a total eclipse) or n (for a partial eclipse) may be found just as in the previous discussion. Only slight changes are necessary in the method of solution previously developed. In the case of a circular orbit equation (40) gives

$$\delta^2 = \cos^2 i + \sin^2 i \sin^2\theta = d_1^2\{\phi(k, a)\} = a_1^2(1 - z\cos^2\theta)\{\phi(k, a)\}^2 .$$

Selecting three values a_1, a_2, a_3 we find, as in § 3,

$$a_1^2[\{\phi(k, a_1)\}^2 - \{\phi(k, a_2)\}^2] = \frac{(\sin^2\theta_1 - \sin^2\theta_2)(\sin^2 i - z)}{(1 - z\cos^2\theta_1)(1 - z\cos^2\theta_2)},$$

and then

$$\psi(k, a_1) = \frac{(\sin^2\theta_1 - \sin^2\theta_2)(1 - z\cos\theta_3)}{(\sin^2\theta_2 - \sin^2\theta_3)(1 - z\cos\theta_1)}. \qquad (43)$$

When $z = 0$ this reduces to the equation (12) previously obtained.

In the case of total eclipse, therefore, we have only to divide each value of $\sin^2\theta_1-\sin^2\theta_2$ by the corresponding value of $1-z\cos^2\theta_1$, and then proceed to find k from the ratios of these numbers exactly as we did previously. The computation of the light-curve when the mean value of k has been obtained involves no difficulty. For the calculation of the elements, if θ' and θ'' correspond to the beginning of eclipse and of totality, we have the equations

$$\left.\begin{aligned} a_1^2(1-z\cos^2\theta')(1+k)^2&=\cos^2 i+\sin^2 i\sin^2\theta',\\ a_1^2(1-z\cos^2\theta'')(1-k)^2&=\cos^2 i+\sin^2 i\sin^2\theta'', \end{aligned}\right\} \quad (44)$$

which give a_1 and i. We then compute

$$\epsilon^2=z\operatorname{cosec}^2 i \qquad b_1^2=a_1^2(1-\epsilon^2) \qquad a_2=ka_1 \qquad b_2=kb_1, \quad (45)$$

and the elements are determined.

In the case of a partial eclipse we may proceed in the same way. The fundamental equation (16) may be written

$$\chi(k, a_0, n)=\frac{\sin^2\theta(n)-\sin^2\theta(1)}{\sin^2\theta(\tfrac{1}{2})-\sin^2\theta(1)},$$

since $\theta(1)=0$. χ is therefore a function of the same nature as the ψ-function, and we have only to divide each value of $\sin^2\theta(n)$ by the corresponding $1-z\cos^2\theta(n)$; so that our equation becomes

$$\chi(k, a_0, n)=\frac{\sin^2\theta(n)\{1-z\cos^2\theta(\tfrac{1}{2})\}}{\sin^2\theta(\tfrac{1}{2})\{1-z\cos^2\theta(n)\}}. \quad (46)$$

We may then proceed as in the case of spherical stars.

When the stars are prolate, and the orbit eccentric, complications ensue; for we must suppose the rotation of the stars to be uniform, while their orbital motion is not so, and the assumption that their major axes are always in the line joining their centers is no longer true. The two will be inclined at a small angle equal to the equation of the center in the elliptic motion. The effect of this inclination on the eclipsed area will obviously be small, and may therefore be neglected. More serious is the probability of tidal effects which may be revealed by a brightening of the light near periastron, or perhaps in other ways. A good approximation to the elements of the system may still be obtained by neglecting

these irregularities, rectifying the light-curve as above, and determining the eccentricity and other elements as outlined for spherical stars—being careful to use the equations (43) or (46) instead of (12) or (16).

PART V. REFLECTION, DARKENING AT THE LIMB, ETC.

§ 9. *Reflection (or radiation-effect).*—The preceding discussion affords a complete solution of the problem of eclipsing variables, so long as we assume that the individual stars appear as uniformly illuminated disks of constant surface-brightness. This assumption is, however, improbable. It is more than likely that the actual stars of a close binary pair appear, like the sun, to be brighter at the center than the limb, and that each star is brighter on the side which is heated up by its companion's radiation than on the other. Direct evidence of the last phenomenon has been found by Dugan[1] in the cases of *RT Persei* and *Z Draconis*, and by Stebbins[2] in that of *Algol*. There are few if any other stars whose light-curves between minima have been studied with anything like the care bestowed upon these three, and it may well be that observations of comparable accuracy will show the phenomenon to be general.

It is probable, as Dugan points out, that in this case the excess of brightness is by no means uniformly distributed over the hemisphere of each star which faces its companion, but is greatest at the center of this region. As the whole amount of the increase in brightness due to this "reflection" effect is but a very small percentage, this variation in its amount is probably less than the general (and so far disregarded) darkening at the limb. Unless we wish to take the latter into consideration, we may follow Stebbins and treat the stars as exhibiting two hemispheres of unequal brightness—the fainter facing away from the companion. To all practical intents and purposes only the brighter hemisphere of each star undergoes eclipse.

[1] *Publications of the Astronomical and Astrophysical Society of America*, **1**, 311 (date of actual publication, 1908), and **1**, 320, 1909.

[2] *Astrophysical Journal*, **32**, 200, 1910.

If the light we would receive from the fainter side of the larger star is l_1-c_1 and from its brighter side l_1+c_1, the rotation of this star uneclipsed would give us a variation expressed by the equation

$$l=l_1-c_1 \sin i \cos \theta ,$$

the minimum occurring when $\theta=0$ and the companion is behind this star. For the smaller star we have in the same fashion

$$l=l_2+c_2 \sin i \cos \theta .$$

Hence, for the light of the whole system, we have

$$l=l_1+l_2+(c_2-c_1) \cos \theta . \tag{47}$$

If we assume that the quantities c, which are proportional to the excess of light radiated by each star on the side toward its neighbor, are also proportional to the energy received by the surface of each star from the radiation of its companion, it follows that these quantities (being proportional to the product of the superficial area of the first star by the total light-emission of the second) are in the inverse ratio of the surface-brightness of the two stars. If the orbit is circular, the last ratio is that of the loss of light at the two minima, and we have

$$c_1(1-\lambda_1)=c_2(1-\lambda_2) , \tag{48}$$

where λ_1 and λ_2 are the light intensities at the two minima. The brightness of the system is therefore greater just outside the secondary minimum than just outside the primary—which is in accord with observation. When the two minima are of equal depth no such effect is to be expected, and none has been observed. Observations accurate enough to detect an effect of this sort will usually indicate also a small ellipticity of the stars. As the influence of the latter is symmetrical about the point half-way between the minima, and the former changes sign there, there is no difficulty in separating them. In fact, as the influence of "reflection" is so small, we may write at once, for its effect on the observed stellar magnitude

$$\Delta m=-1.08(c_2-c_1) \sin i \cos \theta , \tag{49}$$

and determine it by comparison of points on the light-curve, outside eclipse, for which cos θ has values numerically equal but

opposite in sign. Having determined $(c_2-c_1)\sin i$ (in practice we can always set $\sin i=1$), we may find c_2 and c_1 by (48).

If the fainter sides of both stars were increased in brightness till they equaled the brighter hemispheres, we should have disks of uniform brightness. As the fainter sides are never eclipsed, we may make this correction very simply by transforming the observed magnitudes into light-intensity and adding the quantity $c_1+c_2+(c_1-c_2)\cos\theta$ to the observed intensity. The resulting light-curve may now be discussed as usual.

When the stars are decidedly prolate, the assumption that the surface of each is divided into two regions of different but constant intensity would lead to more complicated expressions. But as we are dealing with an approximation in any case, it is most convenient, and accurate enough, to assume that the simple formulae given above shall hold good—"rectifying" the curve first, to remove the effects of ellipticity, and then proceeding as above.

In certain cases, where the orbit is eccentric, the observed brightness of the system may be different at periastron and at apastron, owing to tidal action. The effect, being small, may be represented approximately by a simple harmonic term[1] of the form

$$\Delta m=a\cos\theta+b\sin\theta\,.$$

The first part of this fuses with the "reflection" effect already discussed. The term $b\sin\theta$ gives rise to inequality in the magnitude at the two maxima, and may be determined from such a difference, if it exists. For further computation it is sufficient to remove this term from the observed magnitudes (which practically amounts to taking the mean of the observed light-curves at the two maxima as a normal curve for both).

§ 10. *Darkening at the limb.*—We must finally consider the influence upon our methods and results of the assumption that the stars do not appear as uniform disks, but are darker at the limb than at the center. To determine in detail the actual law according to which the brightness of the disk varies from point to point is obviously too much to expect. We may, however, assume a law (involving as few constants as possible) and require

[1] A. Roberts, *Monthly Notices*, **66**, 127, 1906.

the determination of these constants as part of our problem. Schwarzschild[1] has shown that on certain plausible assumptions the apparent brightness J of the disk, at a point whose normal makes an angle i with the line of sight, should vary as $1+2\cos i$, and that the total energy radiation from the sun's disk nearly follows this law. If we assume the more general law, J varies as $1-x+x\cos i$, where x lies between zero and unity, we shall have an expression capable of representing, approximately at least, the conditions likely to be met with among the stars, and shall have but one more constant to determine from our observations.

The solution, in the case of total eclipse, at least, can be carried out with the aid of new tables, exactly similar in nature to those already discussed, if we let a denote now, not the fraction of area eclipsed, but the fraction of the light of the eclipsed star which is obscured. In the general case a will be a function of the three quantities $\frac{\delta}{r_1}$, k, and x. If the first two of these have fixed values a will obviously be a linear function of x. When $x=0$, it reduces to the function $f\left(\frac{\delta}{r_1}, k\right)$ already calculated. If *i*or $x=1$ we have $a=f_1\left(\frac{\delta}{r_1}, k\right)$ we shall have in general

$$a=(1-x)f\left(\frac{\delta}{r_1}, k\right)+xf_1\left(\frac{\delta}{r_1}, k\right). \tag{50}$$

The new function f_1 may be computed and tabulated, the work being much lightened by simple methods of mechanical quadrature.[2] For selected values of x (say $\frac{1}{3}$, $\frac{2}{3}$, and unity) we may invert the equation (50) and write

$$\frac{\delta}{r_1}=\phi_x(k, a)=1+kp_x(k, a)\,.$$

Constructing new functions $\psi_x(k, a)$ exactly in the manner already employed, we arrive at a method of solution, for any previously assumed value of x, which is formally identical with that already developed, differing only in that different tables of numerical values of ψ are used for the different values of x.

[1] *Göttingen Nachrichten*, Math.-Phys. Kl., 1906, p. 41.

[2] An example of such a method is given by Blazko, *Annales de l'Observatoire astronomique de Moscou*, II Série, **5**, 91, 1911.

The new tables will, however, have to be twice as extensive as the old, for in this case the light-curves caused by the eclipse of a large star by a smaller one will be different from those arising from that of a small star by a larger, and the ψ-table will be different in the two cases. Moreover, during the annular phase of an eclipse the light will not be constant.

Whether the remarkable empirical relations, which serve as a basis for the method already developed for determining the elements in the case of partial eclipse, will hold good in the case of disks darkened at the limb, can be determined only by trial.

When such tables have been constructed, the most probable amount of the darkening at the limb, for any given star, can be determined by comparison of the discordances between the observations and the best light-curves which can be obtained on various assumptions as to the value of x. The most favorable cases will clearly be those where the eclipse is total, as the number of other constants which must be found from the light-curve is then a minimum.

Whatever our assumed value of x, we can make our computed light-curve cross the ascending or descending branch of the observed curve (which we must in all cases assume to be symmetrical), at any three points we choose. The outstanding differences between observation and calculation will therefore always be small, and very accurate observations will be necessary if we are to discriminate between the different values of x.

§ 11. *Influence of darkening at the limb on the computed elements.*— The influence which darkening at the limb will exert on the form of the light-curve, and the nature of the errors which it will introduce into elements computed on the assumption of uniformly illuminated disks, may be easily followed quantitatively with the aid of the function $\psi(k, a_1)$. If the values of $\sin^2\theta$ for two assumed values a_2 and a_3 are A and $A-B$, we have in general for any value of θ, $\sin^2\theta = A + B\psi(k, a)$ and the elements are given by equations of the form

$$\left.\begin{aligned} r_1^2 \operatorname{cosec}^2 i &= \frac{B}{\phi_1(k)}, \quad r_2^2 \operatorname{cosec}^2 i = \frac{k^2 B}{\phi_1(k)}, \\ \cot^2 i &= \frac{B}{\phi_2(k)} - A . \end{aligned}\right\} \quad (51)$$

In the present discussion it is convenient to take for a_2 and a_3 not the values of 0.6 and 0.9, used in computing Table II, but other values, say 0.25 and 0.75, symmetrical about the point where half the light is lost. The necessary transformation is very readily effected, and the results are as follows:

TABLE OF $\psi(k,a_1)$ FOR $a_2=0.25$ $a_3=0.75$

	$k=1.0$	0.8	0.6	0.4	0.2
$a_1=0$	+2.65	+2.41	+2.22	+2.02	+1.87
$a_1=0.25$	0.00	0.00	0.00	0.00	0.00
$a_1=0.75$	−1.00	−1.00	−1.00	−1.00	−1.00
$a_1=1.00$	−1.11	−1.23	−1.35	−1.47	−1.61
$\frac{1}{\phi_1(k)}$	+0.69	+0.82	+1.08	+1.56	+3.12
$\frac{k^2}{\phi_1(k)}$	+0.69	+0.53	+0.39	+0.25	+0.12
$\frac{1}{\phi_2(k)}$	+0.11	+0.20	+0.54	+1.03	+2.56

If the eclipsed star is darkened at the limb, it is clear (assuming a total eclipse) that the middle part of the loss of light will take place more rapidly, and the beginning and end more slowly, than if it was uniformly bright.[1] The same will be true of the variations of $\sin^2\theta$. Since we have taken the middle part of the eclipse as our standard, so to speak, in determining ψ, the values for this function for the beginning and end of eclipse will both be numerically greater than for uniform disks of the same radii. But a glance at the above table shows that, on the assumption of uniformly illuminated disks, larger values of ψ for the beginning of eclipse demand a larger value of k, and numerically larger values near totality, a smaller value of k. It is impossible to satisfy both these conditions. In practice the latter portion of the light-curve must have the preference, because when the star is faint, and θ is small, given changes in a or $\sin^2\theta$ result in much greater changes in the computed stellar magnitude and time than near the beginning of eclipse. We shall therefore find too small a value of k. From the equations (51) and the last three lines of the table, it follows that the computed value of r_2 will be too small, and those of r_1 and cot i too great. Near the beginning or end of eclipse our

[1] S. Blazko, *op. cit.*, p 79.

computations will give too rapid a change of light, so that the observed duration of the change will exceed the computed. Such an effect has been detected by Dugan[1] in the case of *RT Persei*.

The errors in the computed dimensions of the two stars are of course incapable of detection by any known method of control. That in the inclination might perhaps be proved to exist; for, if the eclipsing variables as a class present disks considerably darkened at the limb, there will be more cases in which the computed elements indicate an almost grazing total eclipse (i.e., one in which one star is barely hidden behind the other) than can reasonably be explained on the theory of probability.

In the case of ellipsoidal stars, darkening at the limb presents a complicated problem. As it presumably arises in the atmosphere of the star, and increases with increasing obliquity of the line of sight to the star's surface, the mean effective darkening should be greater when we see the prolate spheroid end on than when it is side on. The variation in light due to the rotation of the star alone will therefore be increased, and the ellipticity computed on the assumption of uniform surface brightness will be too great.

§ 12. *Density of the components.*—It may be appropriate in closing to give the equations which determine the density of the components in terms of the orbital elements.[2] Let the total mass of the system be m, that of the larger star my and that of the smaller $m(1-y)$. If a is the semi-axis major of the orbit, we shall have $a=Km^{\frac{1}{3}}P^{\frac{2}{3}}$, where K is a constant depending on the units of measurement. If we choose the sun's mass, the sun's radius, and the day as units, then for the earth's orbital motion $a=214.9$, $P=365.24$, whence $K=4.206$.

In determining the elements of the system, we have taken a as our unit of length. The actual radius of the larger star is therefore ar_1, and its volume in terms of that of the sun is $K^3mP^2r_1^3$, or $74.4\ mp^2r_1^3$. Its density ρ_1 is therefore $\rho_1=0.01344\frac{y}{P^2r_1^3}$, and similarly, of the smaller star $\rho_2=0.01344\frac{1-y}{P^2r_2^3}$.

[1] *Contributions from the Princeton University Observatory*, No. 1, p. 46, 1911.

[2] A. Roberts, *Astrophysical Journal*, **10**, 308, 1899; H. N. Russell, *Astrophysical Journal*, **10**, 315, 1899.

The actual densities cannot be computed unless the ratio of the masses of the two stars is known. We may, however, make use of the fact that among all the binaries, visual and spectroscopic, in which this ratio has been determined, the brighter star is found to be the more massive (with the doubtful exception of 85 *Pegasi*) for which Boss finds $y=0.36\pm0.13$[1] while the greatest mass-ratio is 3.6 : 1,[2] in the case of the spectroscopic binary *b Persei*. That is, in all known cases y lies between the limits 0.36 and 0.78 for the brighter star, and 0.64 and 0.22 for the fainter.

If we assume $y=0.5$ for both stars, our formula becomes for both stars

$$\rho=\frac{0.006672}{P^2r^3}. \qquad (52)$$

This is likely to give too high a density for the faint star, and too low for the bright one, but in neither case is the error at all likely to exceed 50 per cent of the computed values, or to be in the opposite sense from that stated. We may therefore, in spite of this uncertainty, draw fairly accurate conclusions regarding the density of these stars.

When the stars are ellipsoidal, our formula obviously becomes $\rho=\frac{0.006672}{P^2abc}$ where a, b, and c are the three axes of the ellipsoid. Darwin has shown that in this case the ellipsoidal form of the stars increases their mutual attraction, and shortens the period of the system. This would tend to make the density computed without consideration of this influence, a little too great; but the effect is at most but a small percentage and may be neglected in practice.

A discussion of the light-curves of those eclipsing variables which have been sufficiently observed is now in progress at this observatory. If sufficient evidence of darkening at the limb is discovered to warrant the labor of computing the tables described above, they will be prepared. The computations are in the very efficient hands of Mr. Harlow Shapley, fellow in astronomy, to whom the writer is indebted for valuable aid in preparing the present communication for the press.

PRINCETON UNIVERSITY OBSERVATORY
March 19, 1912

[1] *Preliminary General Catalogue*, p. 278, 1910.

[2] J. D. Cannon, *Journal of the Royal Astronomical Society of Canada*, **5**, 270, 1911.

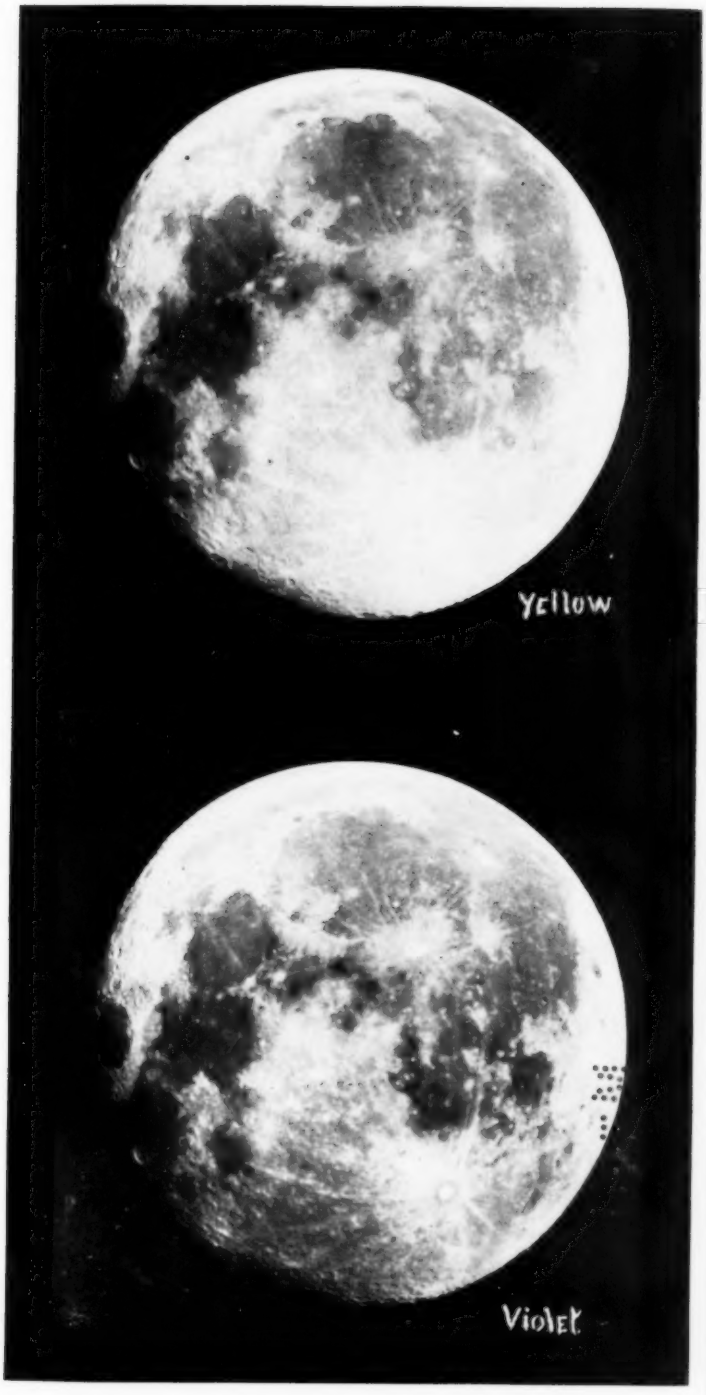
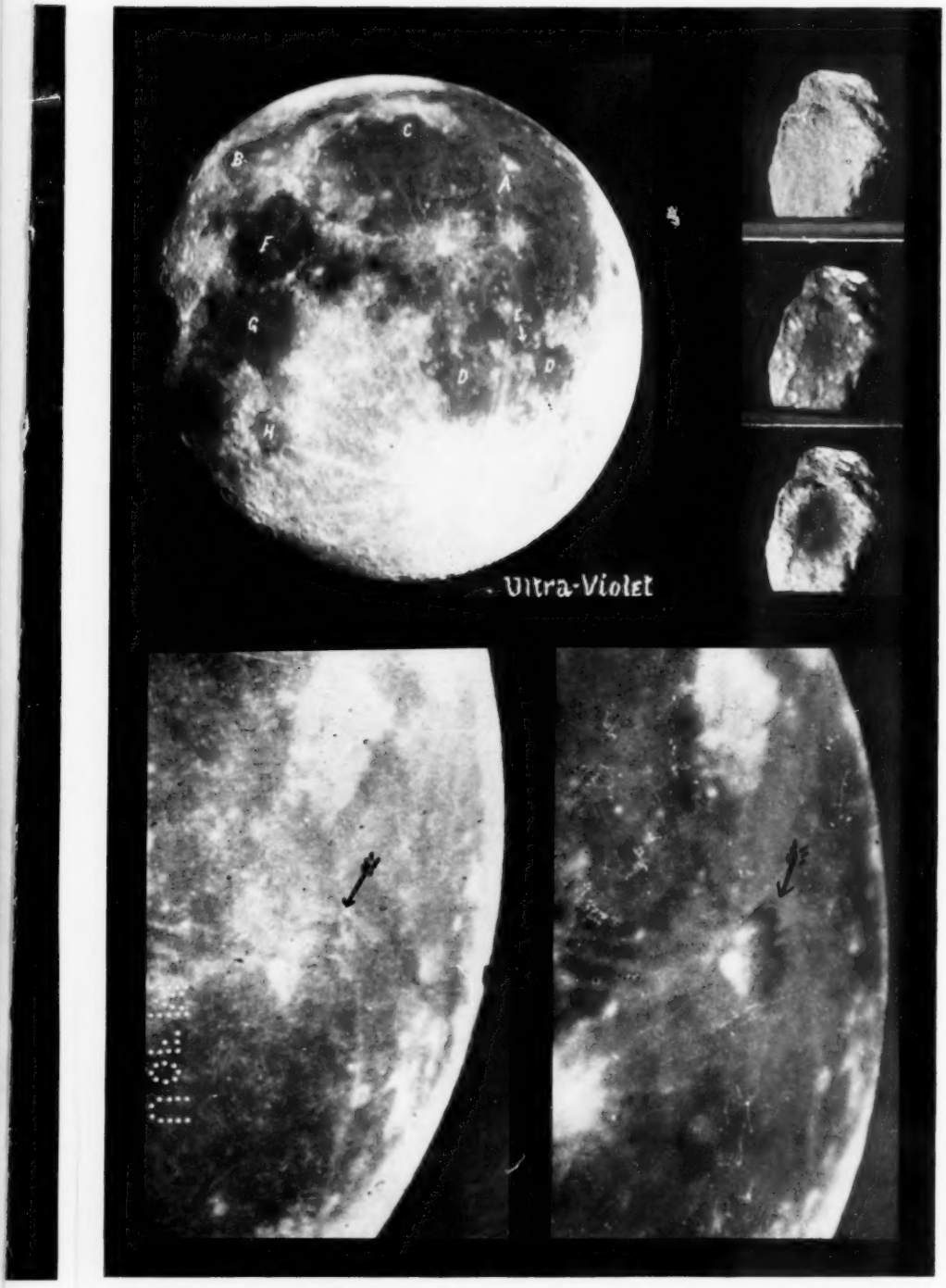

Yellow
Violet

PLATE IX



Yellow

Violet

Ultra-violet

Yellow

Ultra-violet

GRAPHS IN LIGHT OF DIFFERENT WAVE-LENGTHS

SELECTIVE ABSORPTION OF LIGHT ON THE MOON'S SURFACE AND LUNAR PETROGRAPHY

By R. W. WOOD

In a preliminary communication entitled "The Moon in Ultra-violet Light" (*Monthly Notices R.A.S.*, **70**, 226, 1910, and *Popular Astronomy*, **18**, 67, 1910,) I showed that by photographing the moon with light of the shortest wave-length which is capable of traversing the earth's atmosphere, certain features are brought out which cannot be seen or photographed with visible light. The most conspicuous feature was the black area in the vicinity of the crater *Aristarchus*.

In the present paper I shall show that by combining photographs taken by light from three or more regions of the spectrum it may be possible to commence a study of the petrography of the moon's surface. The first experiments were made at my laboratory at East Hampton, Long Island, in the summer of 1909, with a home-made telescope with an objective of quartz, three inches in diameter, coated with a film of silver opaque to visible light, but transparent to the ultra-violet in the wave-length range $\lambda 3160$ to $\lambda 3260$. The telescope tube was mounted on an equatorial mounting improvised from the frame of an old bicycle and provided with slow motion for following in right ascension. Photographs made with this very clumsy outfit showed the dark spot bordering *Aristarchus*, and in the following autumn some better ones were made with a six-inch speculum of about ten feet focus.

In the following year two papers were published by Miethe and Seegert (*Astron. Nachrichten*, No. 4489, 4502, Band 188), who, however, worked with a screen of nitroso-dimethyl-aniline and a silvered glass reflector, and consequently did not utilize the region of the ultra-violet which is not transmitted by glass. They secured pairs of pictures made through an orange ray-filter and the nitroso screen, and by projecting them superposed and in register on a screen through a blue and a red-orange ray-filter obtained a two-color picture which brought out the local differences in the

reflecting power of the moon's surface for the two spectrum ranges in question.

Last summer I took up the work again at East Hampton, setting up a 12-inch equatorial of speculum metal, and ordering a 26-foot glass reflector of 16 inches aperture from John E. Mellish of Cottage Grove, Wis. In the *Astrophysical Journal* for December 1911, I have described the method by which a deposit of nickel was formed on this mirror, for it must be remembered that silver reflects only 4 per cent of the ultra-violet light employed in the work, and is consequently no better than glass. This mirror I mounted in combination with a 20-inch coelostat mirror, also nickeled. The arrangements for following were inadequate, however, and the photographs did not show much more detail than the smaller ones made with the six-inch mirror.

In the late autumn, through the courtesy of the Astronomical Department of Princeton University, I was given an opportunity of mounting the mirror on the 23-inch equatorial. I am under great obligation to Professor Russell for the interest which he took in the work and to Mr. Shapley, fellow in astronomy, who assisted me in handling the telescope and making the exposures. The 16-inch mirror was mounted at the eye end of the telescope and the plate-holder back of the objective, at the edge of the incident beam of rays. The use of the mirror in the Herschellian form of course impairs the definition somewhat with the ratio of focus to aperture which was used. Having had little experience with telescope mirrors, I was of the opinion, when putting the apparatus together, that the definition would be sufficiently good with this arrangement for the mirror in question, and as I was anxious to utilize as much of the light as possible, I adopted it.

The plate-holder was mounted on an old microscope stand turned into a horizontal position, the rack and pinion serving for focusing. Three different regions of the spectrum were utilized. A deep-orange screen used in connection with a Cramer Iso plate furnished the yellow image, which represents pretty closely what we see in visual work. A very short exposure made without any screen gives us what I have designated as the violet image, and a screen made by silvering a sheet of uviol glass 1 mm in thickness,

made by Zeiss, gives us the ultra-violet image. This glass, in thin sheets, exercises no appreciable absorption for the range of wave-lengths transmitted by silver, and answers the purpose quite as well as quartz, which was used in the preliminary work. Careful experiments were made at East Hampton during the summer on the best thickness of silver film. If the film is too thin, much blue and violet light is transmitted, and the picture is made chiefly by these rays. If, on the other hand, it is too thick, nothing at all is transmitted, or the time of exposure is unduly prolonged. Photography of the spark through the screen is no criterion at all, for I found that a film which, with a short exposure, yielded only the group of iron lines in the region λ 3160 to λ 3260, when used for the ultra-violet photography of terrestrial objects illuminated by sunlight, gave pictures which could not be distinguished from ordinary photographs, e.g., a glass jar containing cigars appeared transparent, showing the cigars distinctly. This circumstance is due to the fact that, owing to absorption by the earth's atmosphere, the intensity-curve is falling very rapidly in the ultra-violet. The best way to test a screen is to photograph the solar spectrum with a quartz spectrograph, and increase the thickness of the silver film until no trace of the blue, violet, and upper ultra-violet appears, the narrow band in the region specified alone appearing on the plate. Such a film barely shows the filament of a tungsten lamp, when eye observations are made in a dark room. If the filament appears at all bright the film is too thin. One film which I used failed to show the tungsten lamp at all, though this one was a little thicker than necessary. It is better to have them too thick than too thin, for obvious reasons. The jar of cigars photographed through one of these films appears quite opaque, as if made of black glass.

The ray-filters were mounted immediately in front of the photographic plates. In some cases Iso plates were used for the violet and ultra-violet impressions, and in others Hammer Special and Lumière Sigma. It was found very difficult to guide satisfactorily with the large Princeton refractor, in spite of the fact that the large telescope was used for following. The motion in right ascension was especially troublesome, as it was accomplished by pulling a rope. Considering the large amount of motion in declina-

tion during the three-minute exposures, I am surprised that the pictures turned out as well as they did. An inspection of the three photographs, reproduced in Plate IX, shows that the reflecting power of different areas is quite different for the three spectral ranges utilized. The violet picture is the sharpest, as it was taken with an exposure of a second or less, and the lack of perfect definition is probably the result of the manner in which the plate was mounted, i.e., off the axis. I have lettered certain regions on the ultra-violet picture which deserve special mention. The most conspicuous object is the large dark patch just above the crater *Aristarchus* marked *A*. Practically no trace of this appears in the yellow picture, while it is faintly visible in the violet one. Two enlargements of this region are reproduced, one from the yellow negative, the other from the ultra-violet. Other regions which are relatively dark in the ultra-violet picture will be found near the moon's limb above and to the right of *Aristarchus*, notably the maria *B* and *C*. On the other hand the maria *D*, *H*, and *G* come out relatively darker in the picture made with violet light. The mare *F* is lighter than *G* in the yellow and violet pictures, whereas in the ultra-violet picture they are of equal intensity.

The small crater indicated by the arrow at *E* and the two small craters to the right of it are equally bright in the yellow and violet pictures, while in the ultra-violet, the crater *E* is much darker, almost disappears in fact.

It is very important in work of this kind to be sure that imperfections on the ray-filters or plates are not responsible for minute differences. A very slight variation in the thickness of the silver film will cause spurious effects, of course. I have verified the points mentioned on a number of negatives taken with the screens in different positions, which is the only way to distinguish between the real and the spurious.

It is obvious that the greater the number of widely isolated regions of the spectrum which we make use of in photographing the lunar surface the greater are the possibilities of drawing conclusions as to the nature of the materials of which the surface is composed. At the present time we can photograph only to wavelength λ 8000 or thereabouts, but if we could extend the range say

to 8 μ, where anomalies in the reflecting power are shown by the silicates, we should probably find ourselves in the position to take up the subject of lunar petrography. Even with the range capable of investigation at the present time we can probably make a beginning, as I shall illustrate by a study of the *Aristarchus* dark spot, which, as I have said, is the most conspicuous object brought out by the ultra-violet photography. This spot is invisible in yellow light, begins to appear faintly in violet, and is very dark in ultra-violet.

I made a series of photographs with the three spectrum ranges used in the lunar work of a number of volcanic rocks. In this way I selected two specimens of volcanic tuff, of about the same color, one of which photographed dark in ultra-violet light. Superposing a small chip of this specimen on the other reproduced the conditions of the *Aristarchus* spot almost exactly. In yellow light the small chip was almost invisible against the larger, in violet it could be seen to be a trifle darker, and in ultra-violet it was very much darker than the background. I then analyzed the small chip and found that it contained iron and traces of sulphur. Photographs were now made of rocks with surface stains due to iron, but the stains were equally dark in violet and ultra-violet light. I then formed a very thin deposit of sulphur in a spot at the center of a fragment of that specimen of the tuff which photographed light in ultra-violet. The deposit was absolutely invisible to the eye, and was formed by blowing a small jet of sulphur vapor against the surface for a second or two. I photographed the specimen with the three types of radiation, and found that the spot was invisible in the yellow picture, gray in the violet one, and quite black in the ultra-violet. The three pictures are reproduced. The large area and the general form of the *Aristarchus* spot suggested to me, when it was first discovered, that it was made of some material thrown out by a volcanic blast from the crater. It may be an ash containing sulphur or it may be a deposit of sulphur formed by condensation of ejected vapor. It seems probable, however, that it is due to sulphur. Zinc oxide I have found to be quite black in ultra-violet but it is quite as light in violet light as in yellow; therefore the spot cannot be zinc oxide, which would be less probable than

sulphur, aside from the evidence cited. It is apparent that the introduction of the violet pictures has improved the method of analysis, and a further increase is desirable. By means of the infra-red screens, with which I have obtained most remarkable photographs of sunlit foliage showing snow-white against a sky as black as midnight, we can make photographs of the moon in the spectrum range λ 7000 to λ 7400, while the nitroso screen will give us pictures in the upper ultra-violet. With a series of five pictures made with five ranges of the spectrum between wave-lengths λ 7400 and λ 3160, it is probable that new facts would come to light, especially if the lunar pictures were made on a larger scale with an instrument capable of accurate following.

Light within the spectrum range λ 7000 to λ 7400 is but very slightly scattered by the earth's atmosphere, which accounts for the intense blackness of the sky in photographs made by light of this wave-length, and it does not seem impossible that photographs of the brighter planets made through an infra-red screen might prove interesting if the planets are surrounded by a light-scattering atmosphere, for we must bear in mind that the surface of the earth, as seen from a neighboring planet, would be seen through a luminous haze, equal in brilliance to the blue sky on a clear day, that is, it would present much the same appearance as is presented by the moon when seen at noonday.

A solution of cyanine and a yellow dye mixed together in such proportion that only the range λ 7000 to λ 7400 is transmitted makes an even better screen than the one which I used originally, as has been observed by Pfund.

The plates obtained through the ray-filters can be studied to advantage by the methods employed in the three-color process of color-photography. Mr. F. E. Ives, the pioneer in work of this kind, was good enough to make at my request a three-color lantern slide from the three negatives. The negative taken through the ultra-violet screen was printed on a gelatin film and stained blue, the violet and orange pictures being rendered in red and yellow respectively. The three films when superposed resulted in a very pretty color photograph which brought out the differences in the reflecting power of the different maria in a very striking manner.

The prevailing tone of the darker portions of the lunar surface was olive green but certain spots came out with an orange tone and others with a decided purple color. The dark spot near *Aristarchus* came out deep blue, as was to be expected.

I have applied this same process to the two photographs of the *Orion* nebula taken by Professor Hartmann and published in this journal some years ago. One of these was taken through the nitroso-dimethyl-aniline screen which I described in the *Astrophysical Journal* for 1903, and the other through a screen which excluded the ultra-violet light. The marked localization of the two gases which make up the nebula is brought out most beautifully in the resulting color-picture. It is my plan next summer to begin work on photographing stars through the silver ray-filter, for it seems probable that high-temperature stars will come out relatively very bright when photographed with the shortest wave-lengths.

Great care is necessary in the preparation of the silver ray-filter. I find that the uviol glass must be handled very carefully in the acid and alkali cleaning solutions. One of my plates, after having been cleaned a number of times, refused to take a uniform deposit, the silver coming down in streaks and cloudy patches, some parts of the surface refusing to take any deposit at all. More vigorous cleaning only made matters worse, and I found on washing and drying the plate that the surface had become badly corroded, resembling very old window glass which has been exposed to the weather. Repolishing with rouge is the only thing to do when this trouble appears. The silver film must be of uniform thickness, free from even the minutest pin-hole, and appear structureless by transmitted light. This last matter can be best examined into by placing a sheet of velox paper in a plate-holder, covering the holder with the silver filter, and exposing for a few seconds to the light of the blue sky through an *open* window. The window should be at some distance in order that patches of irregular thickness, if they exist, may throw sharp shadows. On developing the paper any bad spots in the film become at once apparent.

As silver films of sufficient uniformity to serve as ray-filters require special precautions in their preparation a few words on this

subject may not be out of place. The formaldehyde process gives the best results apparently, but it is very tricky to work with, and extremely sensitive to proportions, dilution, temperature, etc., acting in three or four wholly different manners according to circumstances. Commercial formaldehyde is diluted with ten parts of distilled water to form the reducing solution. Personally I never weigh my nitrate of silver as I enjoy the element of the personal equation, which enters the problem when scales are dispensed with. From one to two grams to 100 ccm of distilled water are about right. Add ammonia diluted with several volumes of water a little at a time until the precipitate is nearly but not quite dissolved. If too much is added and the solution clears add nitrate of silver solution, a drop at a time until a permanent straw tint is produced. The addition of the ammonia should form a good brown precipitate. Sometimes a very thin precipitate forms, the solution appearing pink or blue by transmitted light. I attribute this to too dilute nitrate solution, and find that it does not work as well as when the turbid brown precipitate forms. The plates carefully cleaned with nitric acid and caustic potash, and thoroughly washed with tap water, lifting them several times to get rid of the potash solution which lurks beneath them, are now flowed with a little of the reducing solution, just sufficient to wash over the surface when the dish is shaken. The dish is now tipped to one side and an equal amount of the silver solution is added, taking care not to pour it onto the plate but to one side. If the plate fills the dish it is better to mix the solutions in a beaker and then flow them over the plate. If the proportions and temperature are right the solution will turn pink in half a minute or so, the color deepening rapidly, and a beautifully uniform blue film will deposit. The process can be repeated until the film has the required thickness. I usually mount a tungsten lamp on the floor so that I can watch it through the bottom of the dish. Unfortunately things as described above seldom happen at the first trial. The silver may come down almost immediately, in brilliant streaks and splotches, the solution remaining clear. This means that too much of the silvering solution has been added. Or things may commence all right but the deposit may be brown like lightly

smoked glass, the solution changing from pink to gray with a distinct precipitate. These brown deposits I find are non-conducting and are probably made up of very small particles. They have a low reflecting power. The trouble in this case is too little silver solution for the amount of reducing solution used. Sometimes the solution becomes muddy at once forming a brown deposit. I suspect that this results from too dilute formaldehyde solution. It troubled me much when my personal equation contained one more variable than at present, but I have not seen it occur recently. As the production of the uniform blue film depends upon getting the proportions just right I suppose the beginner had best mix measured amounts for each trial, unless he has access to a large jar of silver nitrate which "belongs to the department." It is impossible to form a film of sufficient thickness with one silvering by this process. I usually pour off about half of the solution when the operation appears to be over and add a little more of the silvering solution. This usually doubles the thickness. For a second silvering mix the solutions quickly in a clean dish, using a little more of the silvering solution in proportion to the formaldehyde than for the first coating. Transfer the plate to this without washing. If it is washed, the silver is apt to frill in the second bath. The cause of this I do not know. Probably it is osmotic or perhaps catalytic (!).

I feel quite certain that the method of studying the lunar surface outlined in this paper is worth going on with, but only with the largest instruments, perfectly mounted, and under the best atmospheric conditions. The exposure factors for the screens should be carefully determined in advance by photographing well-lighted white paper through them (strip exposure by drawing slide out by degrees) and developing the plates simultaneously. The work can be done to advantage only by one who has had much experience with photographic plates, for it is no easy matter to get three images of the moon through the three specified screens all of which are properly timed and developed to exactly the same density. Unless this is done, very erroneous conclusions will be drawn, and I am not at all sure that my own pictures are wholly free from such defects.

Though a nickel-plated reflector is necessary if the silver ray-filter is to be used, I believe that results of interest can be obtained

with an ordinary silver-on-glass reflector, using a nitroso screen in place of the silver film. This gives us an ultra-violet picture made with slightly longer waves than the ones which traverse silver, but I have found that the spot near *Aristarchus* shows well with one of these screens.

I shall be glad to lend my thin plates of uviol glass to any observatory at which it is planned to take up the work with a nickeled reflector. The plates are 12 cm square and 1 mm in thickness, and I shall be glad to silver them anew with films of the most suitable thickness.

I have been aided in this investigation by a grant from the Elizabeth Thompson fund, made several years ago, and this communication is the final report.

JOHNS HOPKINS UNIVERSITY
May 1912

Reviews

Outlines of Applied Optics. By P. G. Nutting. Philadelphia: P. Blakiston's Son & Co., 1912. Pp. 234. Figs. 73. $2.00.

This book might be called a miniature encyclopedia of the applications of optics in connection with physical instruments. Primary emphasis is placed on the matter of relative and absolute precision and sensibility, in relation to the design and manipulation of optical instruments and the interpretation of their performance. In an introduction and twelve short chapters it contains a wealth of information and suggestion presented in concise fashion, at times so condensed as to seem like a kind of syllabus.

The introduction, after some initial remarks on sources of light and varieties of spectra, gives a preparatory sketch of the various physical properties of light, quoting as needed a few quantitative laws of emission, reflection, refraction, absorption, and polarization, without proof but with some explanation of their significance.

Geometric optics in the ordinary sense is given three chapters which discuss respectively the theory of formation of images, the designing and testing of optical systems, and the principal forms of complete instruments. The theoretical basis here includes the Gaussian theory, the Seidel classification of aberrations of lowest order, and the listing of the conditions to be imposed in the designing of particular instruments, together with the necessary limitations and compromises occurring in connection with each. This treatment is paralleled by a description of methods of determining how closely any actual instrument meets theoretical expectations. A fourth chapter is devoted to methods of determining indices of refraction, to the influence of physical conditions on index, and to representation by dispersion formulas.

Two chapters, on the eye and vision and on colorimetry, treat of the geometric and physiological conditions of visual perception, of photometric and chromatic sensibility, and of the artificial instruments used in the empirical analysis of color. Three chapters deal with sources and distribution of illumination, with photometry, and with the analysis and measurement of radiation as a flux of energy; then three more with the uses of polarized light, the physical properties of the photographic plate, and the metrologic aspect of interference.

The table of contents is full enough to serve fairly as an index, and each chapter is followed by a short bibliography, partly of general references, partly of papers bearing on items in the text.

There are a number of misprints, most of them fortunately obvious, and some errors in single statements. For example, the ordinary eyepiece should be said to be designed for oblique, not axial, achromatism (p. 86), and the separation of lenses in the Clark objectives to introduce rather than eliminate lateral chromatic errors (p. 88). Gauss points and nodal points are confused on p. 33 but distinguished in the specifications of the eye (p. 116). The enumeration of algebraic conditions on p. 53 is faulty. Moreover, the extreme condensation found necessary has sometimes led to the insertion of statements which are obscure and liable to erroneous interpretation. Thus, it is said that the sine-condition and the condition for elimination of coma are equivalent, which is true only for systems free from spherical aberration (p. 54). The convenient distinction of "pupils" and "windows" seems not always consistently observed. It is hard to see why a magnifier of moderate power is recommended for testing objectives (p. 67), or in what sense the effective pupil of the eye can be said to be in front of the cornea (p. 117); or why known radiation laws are called empirical, when their theory is so prominent in the recent history of physics.

It is apparent, however, that the task of compressing such a range of material into the space chosen can have been no easy one. This may account for the difficulty in securing uniform precision of statement, as well as for the omission of some topics which could rightly claim a place; for instance, a direct study of the spectroscope, or Abbé's discoveries regarding the interpretation of microscopic images. The justice of perspective, amount of really valuable imformation, and enthusiastic spirit of the book show that its composition was in good hands. It is surely to be hoped that it may prepare the way speedily for a more ample treatise of similar scope, such as is hinted at in the preface.

A. C. LUNN

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